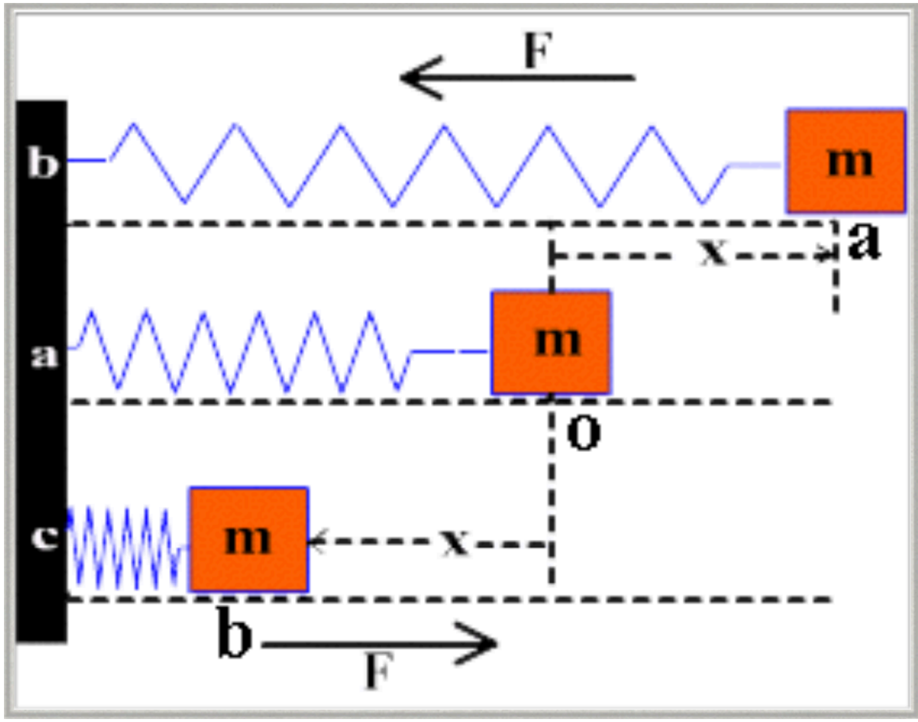
Simple Harmonic Motion

- Oscillatory motion under a restoring force proportional to the amount of displacement from equilibrium
- A **restoring force** is a force that tries to move the system back to equilibrium
- For a spring-mass system like the one on the right, the restoring force is the spring force



- Important because SHM is a good model to describe vibrations of a guitar string, vibrations of atoms in molecules, etc.

Spring force

$$F_s = -kx$$

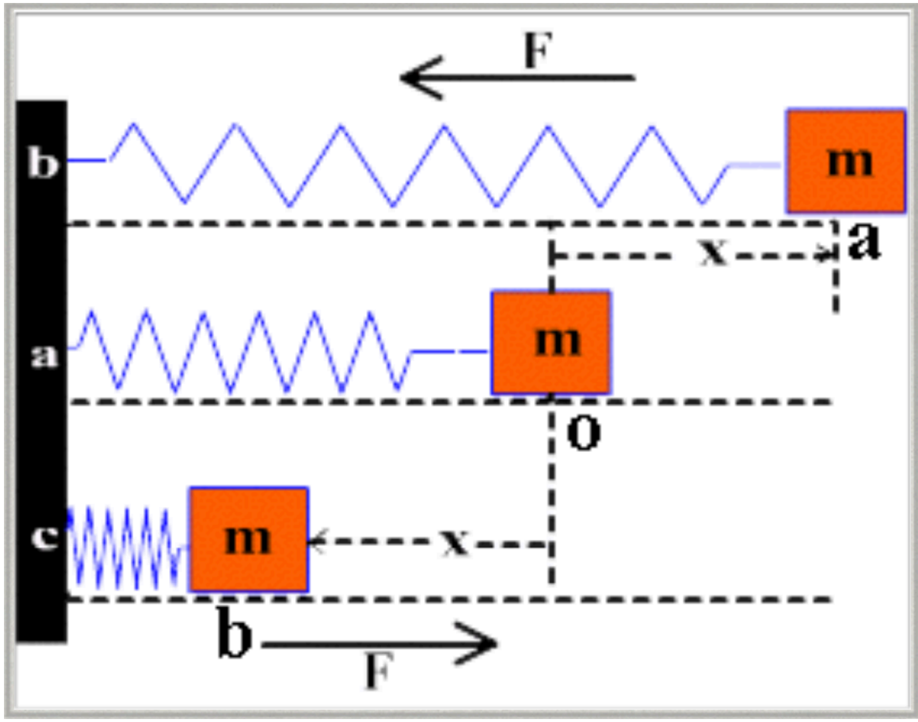
Spring potential energy

$$PE_s = \frac{1}{2}kx^2$$

K = spring constant, measure of stiffness of spring

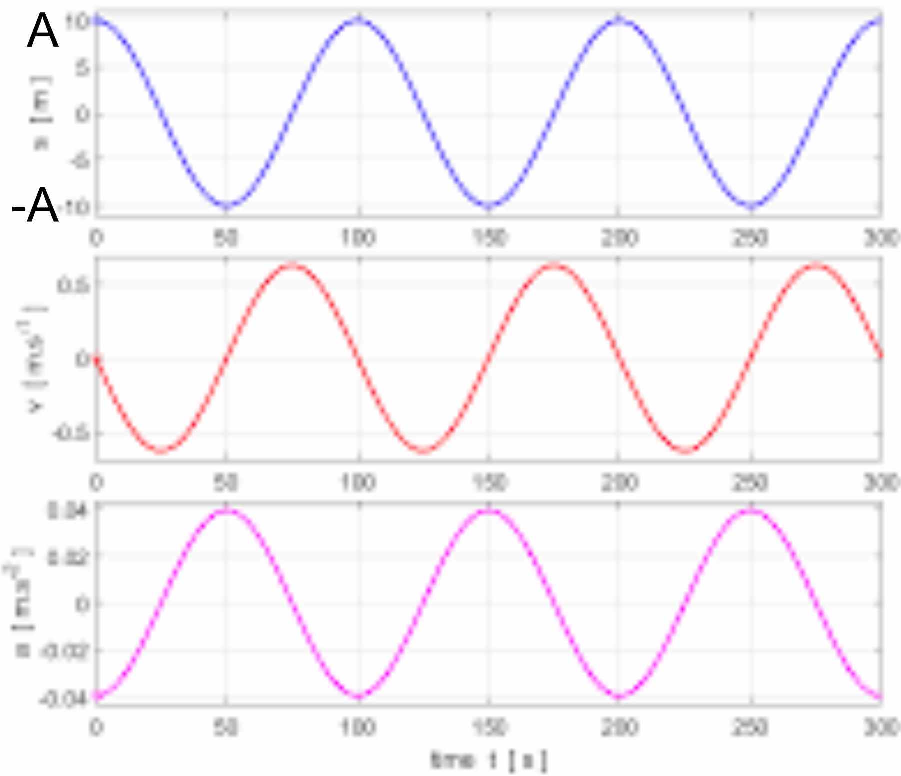
Assumption: **No friction and massless spring**

$$a = \frac{F}{m}, \qquad a = -\frac{kx}{m}$$



A

-A

- 

$$x(t) = A\cos(\omega t)$$

$$v(t) = -\omega A\sin(\omega t)$$

$$a(t) = -\omega^2 A\cos(\omega t)$$

Energy in SHM

$$E_{tot} = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$v(x) = \sqrt{\frac{k}{m}(A^2 - x^2)}$$

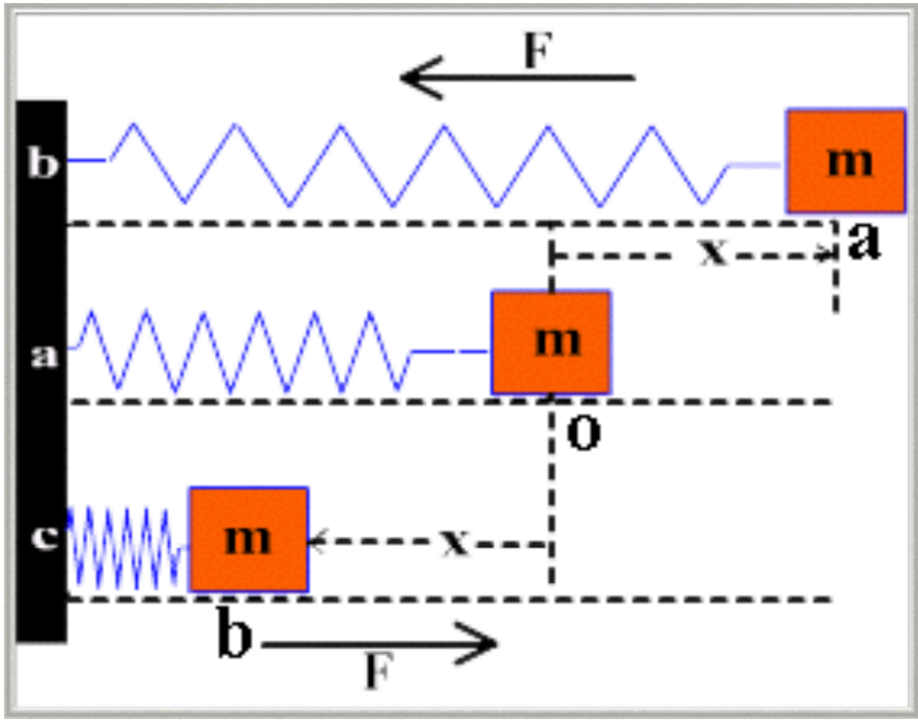
$$x = 0, v = max$$

$$v_{max} = \sqrt{\frac{k}{m}(A^2 - 0)} = \omega A$$

Displace mass to x = A and let go

Fill in the blank:

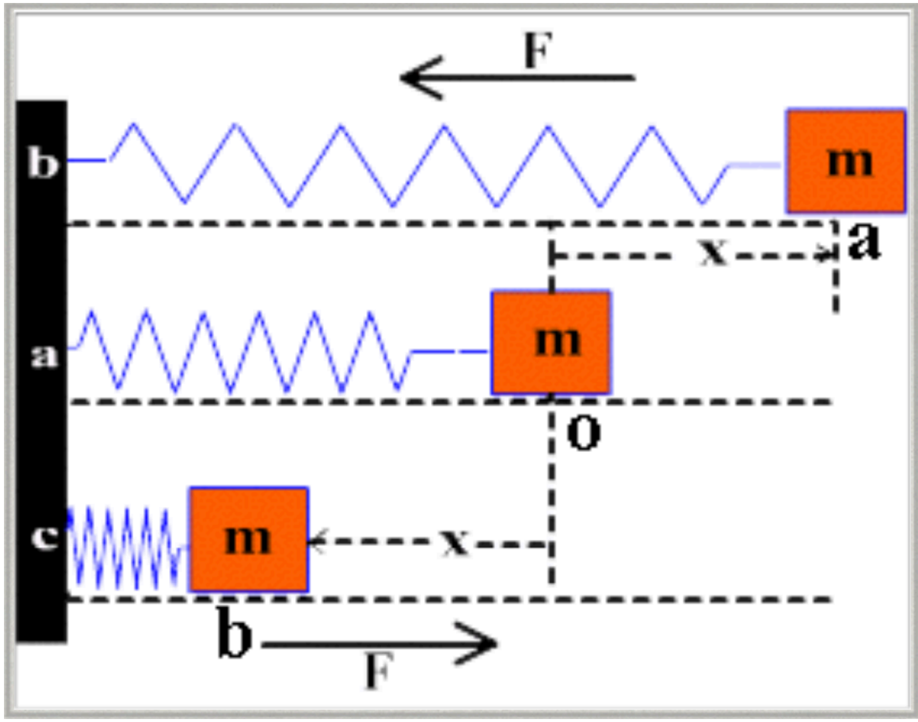
- Net force and acceleration are toward which direction? _____
- Max velocity is at x = _____
- Net force at *O* is ____
- Overshoots and compresses spring to x = ______
- Net force and acceleration are toward _______
- Max KE and max velocity at point 0 (equilibrium)
- Max PE, Max force, max acceleration, KE = 0, v= 0 at point A and B



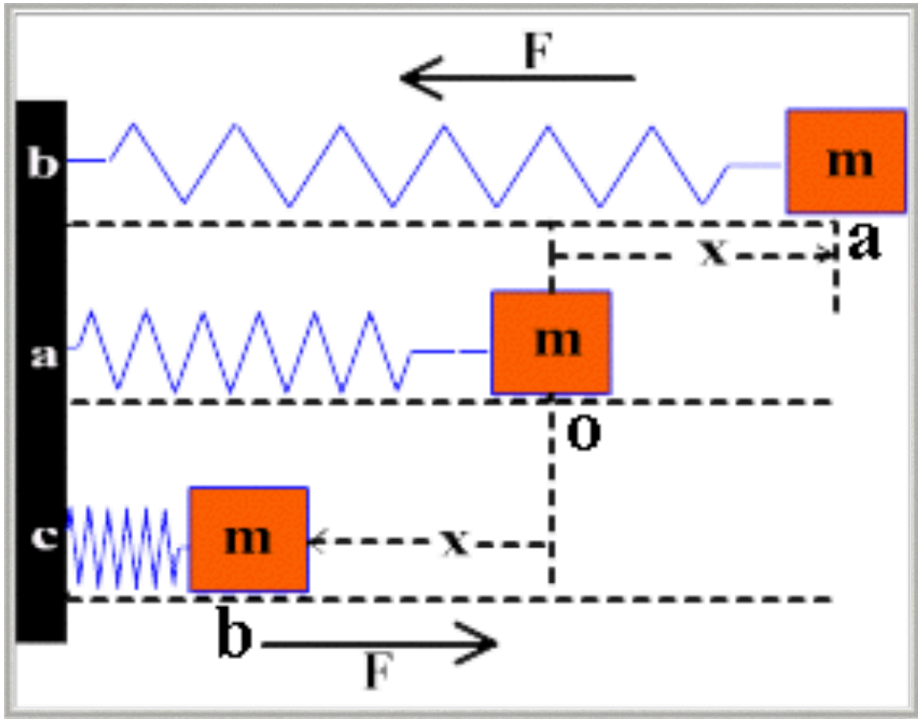
Displace mass to x = A and let go

Fill in the blank:

- Net force and acceleration are toward which direction? __**Left**______
- Max velocity is at x = __**0**___
- Net force at *O* is __**0 N**___
- Overshoots and compresses spring to x = ____**-A**___
- Net force and acceleration are toward __**Right**___

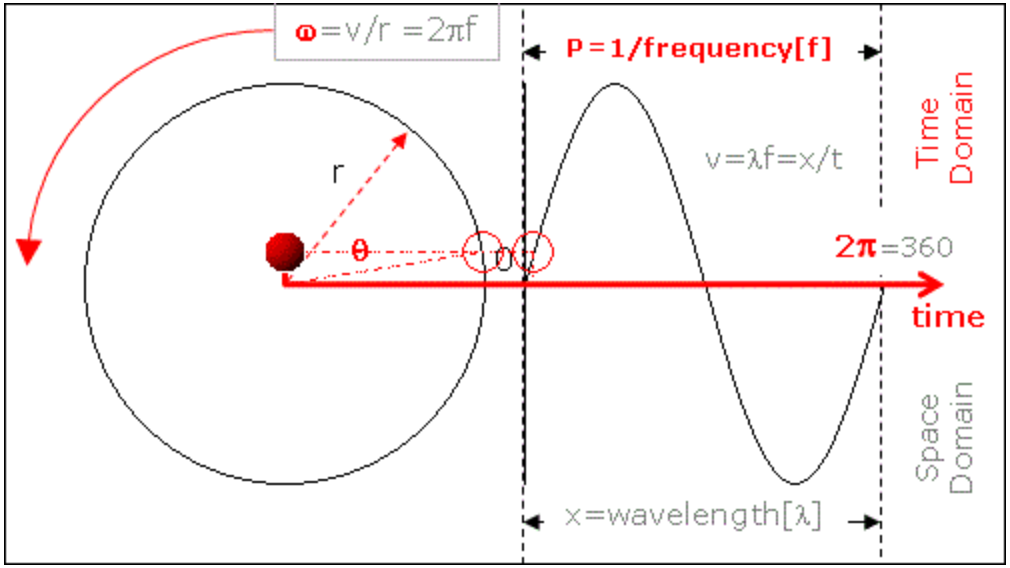


- Amplitude (A) — magnitude of displacement from
- Period (T) — seconds per cycle;
  - $T = 1/f$
- Frequency (f) — cycles per second
  - $f = 1/T$
- Angular frequency (ω) — $\omega = 2\pi f = 2\pi/T$



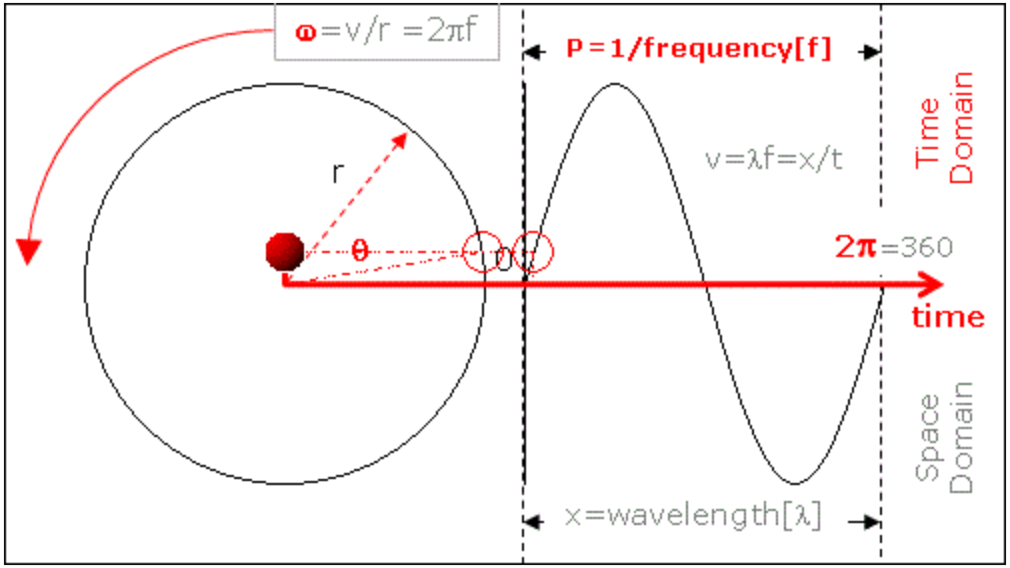
Terms for periodic motion

- Amplitude (A) — magnitude of displacement from
- Period (T) — seconds per cycle;
  - $T = 1/f$
- Frequency (f) — cycles per second
  - $f = 1/T$
- Angular speed (ω) — $\omega = 2\pi f = 2\pi/T$



SHM and Circular Motion

- If the amplitude of the mass's oscillation is equal to the radius of the object in circular motion, and angular speed of the object in SHM = angular speed of object in circular motion
- Their motions are identical



SHM and Circular Motion

- Acceleration of circular motion:

$$a = \frac{v_t^2}{r} = \omega^2 r$$

- Acceleration of spring-mass system:

$$a = \frac{km}{x}$$

Motion is the same, so you can set accelerations equal to each other, and r = x because radius = amplitude (maximum displacement)

$$\omega = \sqrt{\frac{k}{m}}$$

Things to note

- Period and frequency don't depend on amplitude A, even though object is traveling farther with larger A
  - Bigger A = larger restoring Force = higher average velocity

A 17.0 g mass on a 35 N/m spring is pulled 20 cm from equilibrium and released. What is the position of the mass at time t = 1.2s?

- $x = A\cos(\omega t)$
- $\omega = \sqrt{k/m}$
- $\omega = \sqrt{(35\ N/m)/(0.017\ kg)}$
- $\omega = 45.4$ rad/s
- $x = (.2\ m)\cos[(45.4\ rad/s)(1.2\ s)]$
- $x = -0.101$ m or -10.1 cm

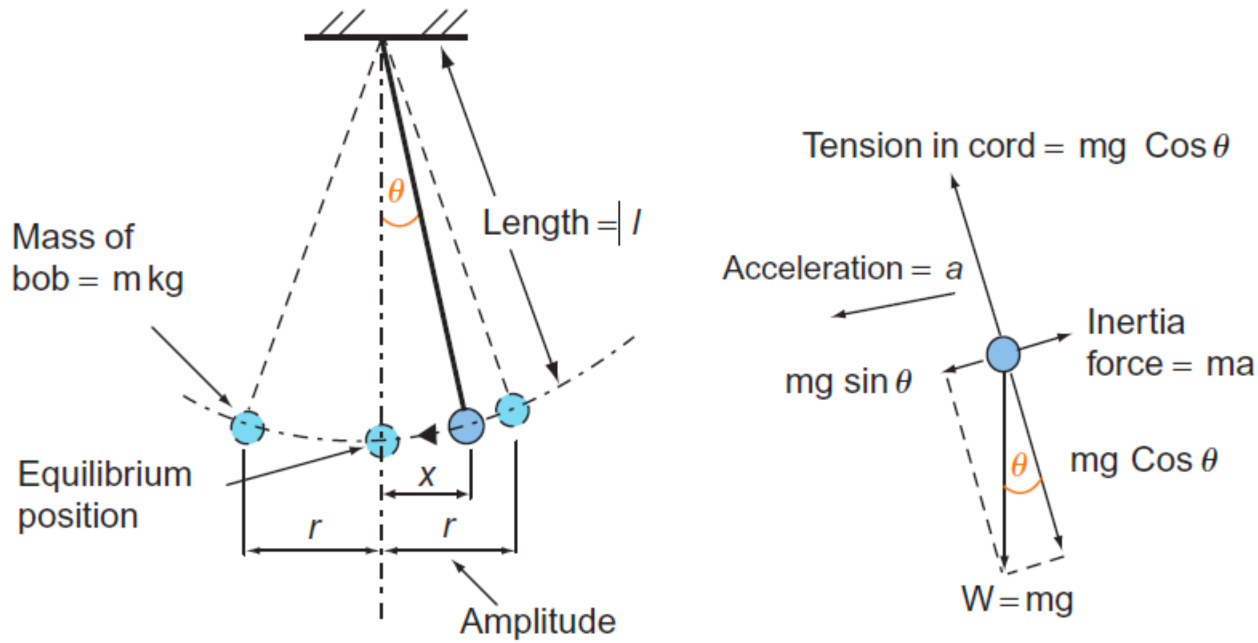
SHM of simple pendulum

- Almost exactly same as spring- mass system, except the restoring force is **gravity**
- $T = 2\pi\sqrt{\frac{L}{g}}$
- Independent of Amplitude (A)
- Independent of mass *m*

Mathematically deriving the pendulum equation

Small angle approximation

- If the angle is small (< 10 degrees), $\sin\theta \approx \theta$



(a) Simple pendulum (b) Forces acting on bob

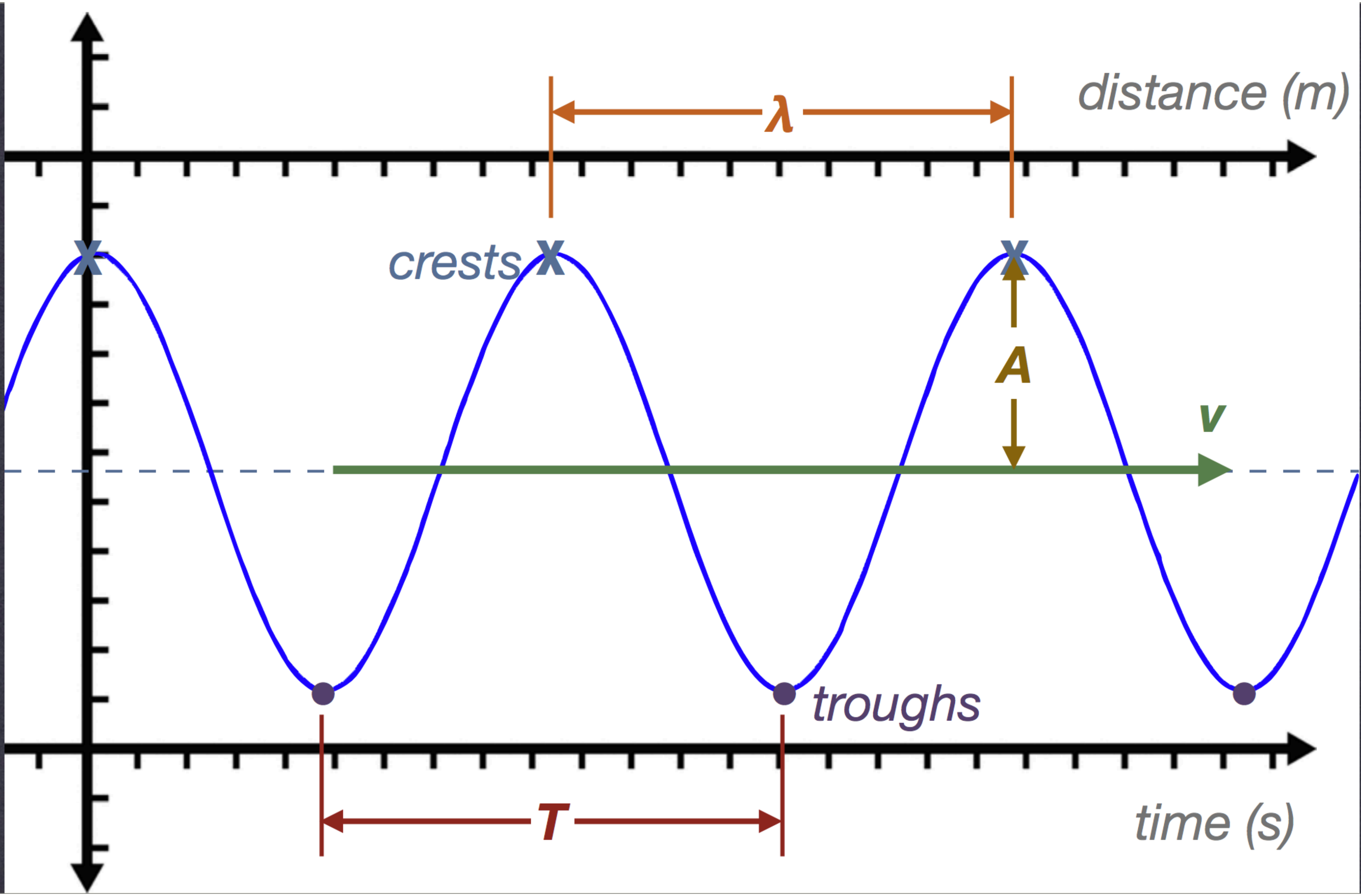
What’s a wave?

- A **wave** is a wiggle in time and space
- The source of a wave is a **vibration**
  - Vibrations are wiggles in time
  - Wave is essentially a traveling vibration

Wave does **not** transfer matter, it transfers **energy**

Qualities of a wave

- **Period (T)** – time it takes for 1 cycle, in seconds (s)
- **Wavelength (λ)** (Greek letter lambda)– distance between successive identical parts of the wave, in meters (m)
- **Frequency (**f) - # vibrations in given time, in Hertz (Hz)
  - $f = \frac{1}{T}$
- **Velocity (v)** – speed and direction of the wave, in m/s
  - $\boldsymbol{v} = \boldsymbol{\lambda} f$
- **Crests** – peaks or high points of the wave
- **Troughs** – low points of the wave
- **Amplitude** –distance from midpoint to crest (or trough), maximum displacement from equilibrium

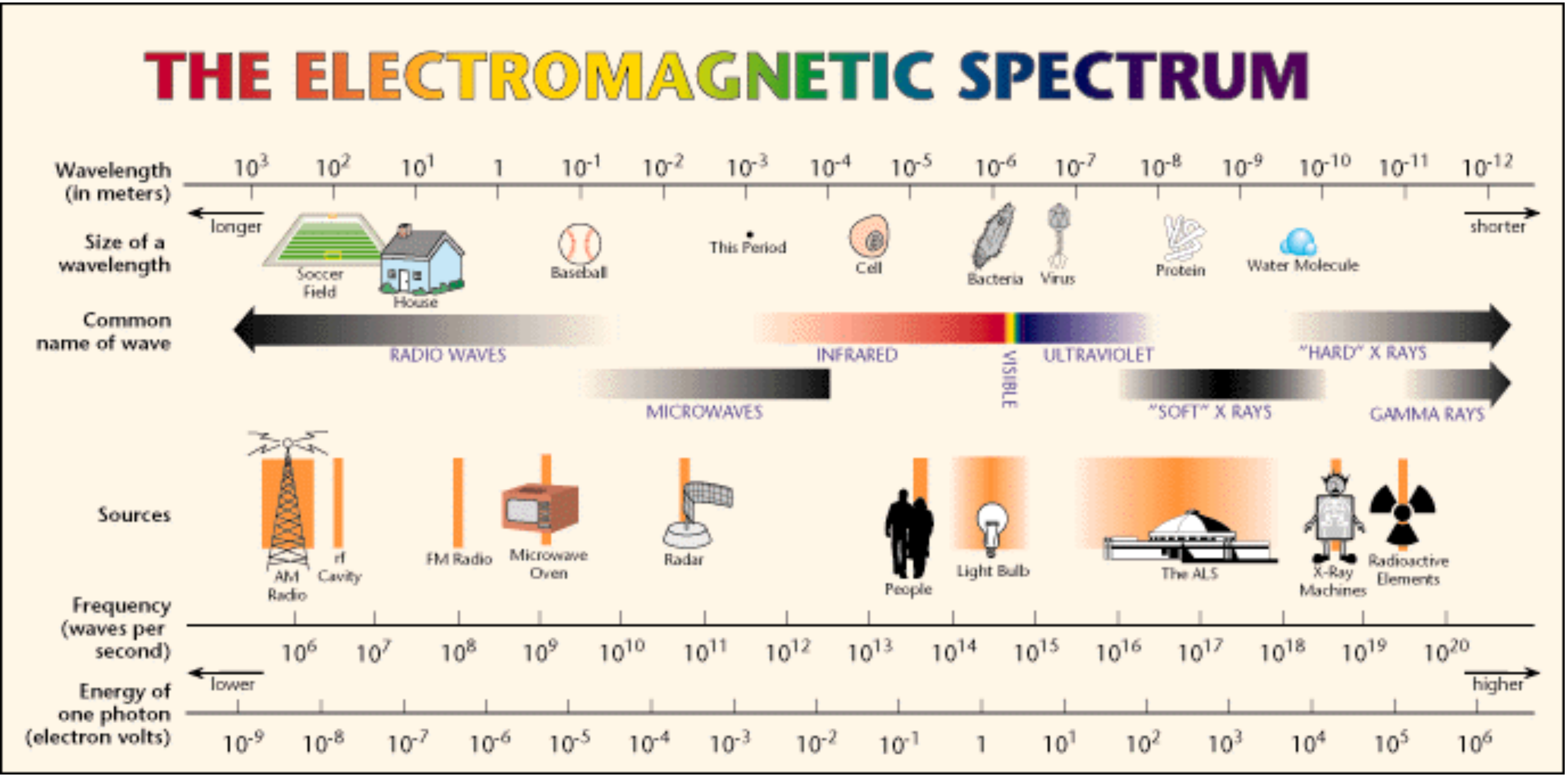

distance (m)
λ
crests
A
v
troughs
T
time (s)

Wave Speed

- In a freight train, each car is 10 m long. If two cars roll by you every second, how fast is the train moving?
  - $v = d/t = 2x(10\ m)/(1\ s) = 20\ m/s$
- A wave has a wavelength of 10 m. If the frequency is 2 Hz, how fast is the wave traveling?
  - $v = \lambda f = (10\ m)(2\ Hz) = 20\ m/s$

- Speed of a light wave
  - c = 3.0 x 10^8 m/s
- Speed of sound (in dry air at 20º C)
  - v = 340 m/s
- Speed of sound in a vacuum (in space)?
  - v = 0 m/s

All waves on the electromagnetic spectrum have a wave speed of 3.0 x 10^8 m/s, they differ in their wavelengths and frequencies.

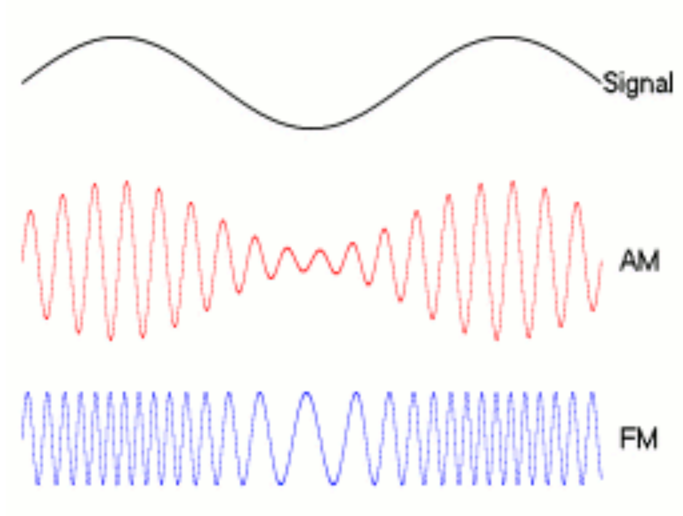


Wave speed depends on medium

- Sound waves travel faster if air is warmer, and travel faster in solids
- Wave speed = elasticity of medium/inertia of medium
- For a wave traveling down a rope
- $v = \sqrt{\frac{F_T}{(\frac{m}{L})}}$
- Tension is restoring force, so greater tension means the rope returns to equilibrium faster, so wave can travel faster.

Waves carry energy, not matter

- **Correction from lecture today: Waves also carry information!**
  - The amplitude, frequency, wavelength, and wave speed are considered information
  - Amplitude modulation (AM) and frequency modulation (FM) are ways to use the information of a radio wave to ”translate” to a sound wave.



Reflection

- When a wave reaches a boundary, it reflects ("bounces") off the boundary
- If it reflects off a fixed point, it also inverts
  - Newton's 3rd law: wave exerts an upward force on the support, support exerts downward force on rope (see simulation)