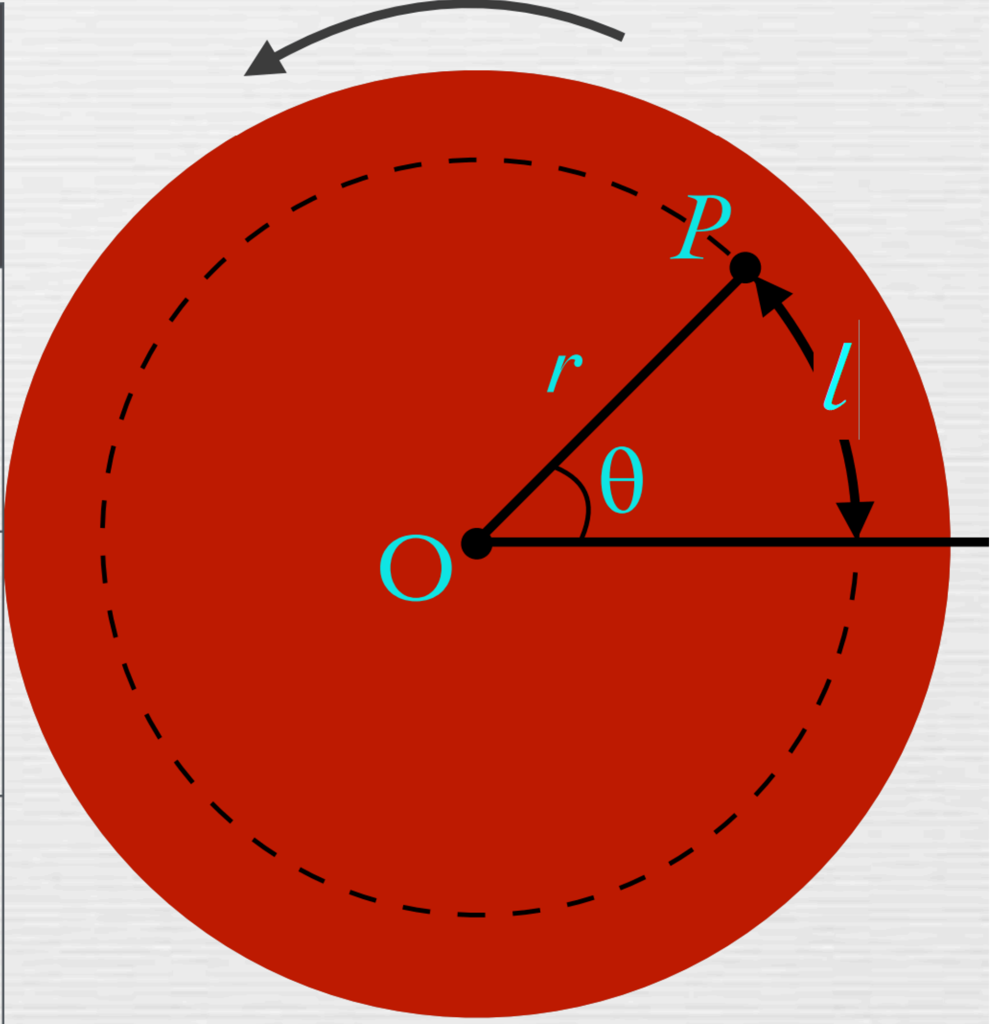
Quantity	Linear	Angular	Relationship
position	l in meters	θ in radians	$\theta = l/r$
velocity	v in m/s	ω in rad/s	$\omega = v/r$ $= \Delta\theta/\Delta t$
acceleration	a in m/s^2	α in rad/s^2	$\alpha = a/r$ $= \Delta\omega/\Delta t$

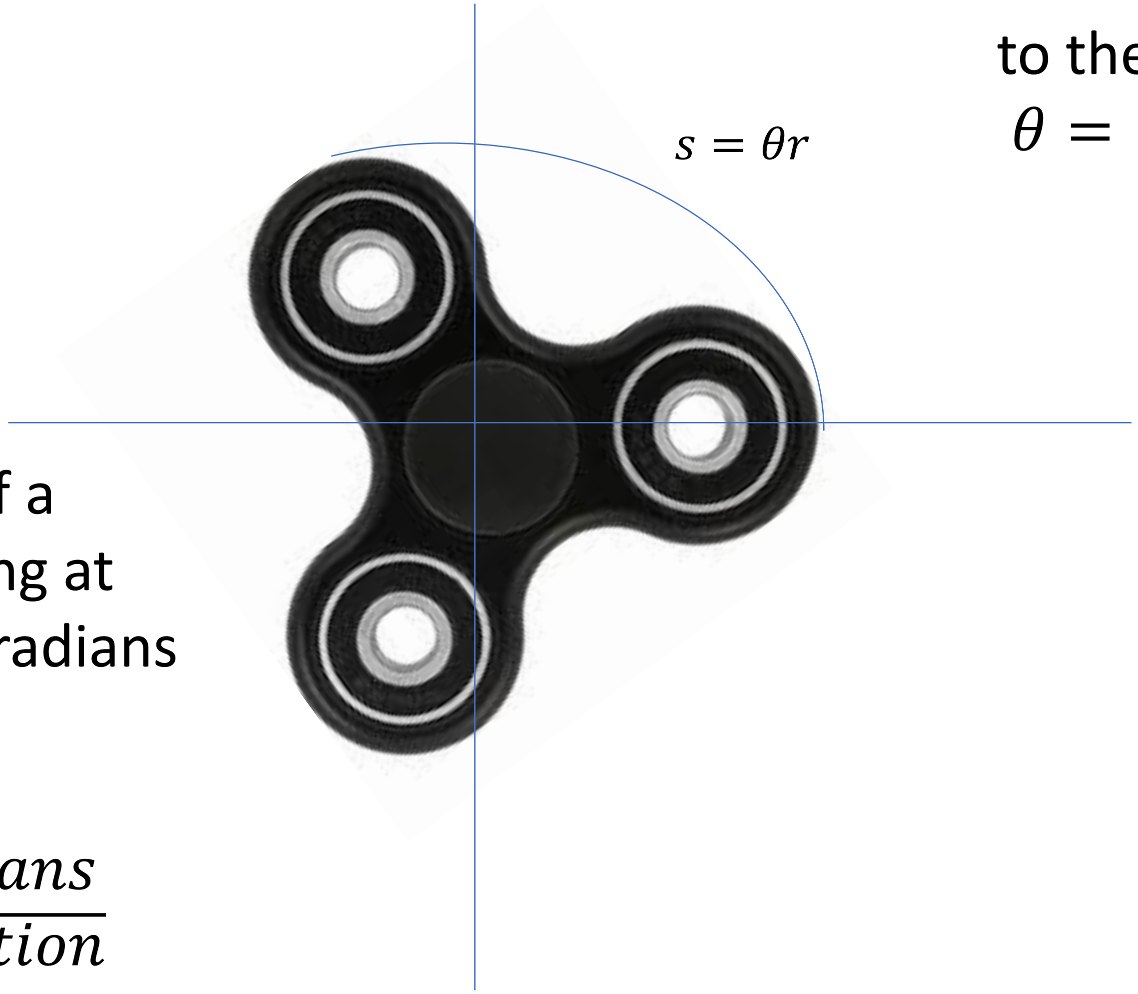


Note: 2π radians = 1 rotation or revolution = 360 degrees. These are conversion factors.

Fidget spinner example

If 1 rotation of the fidget spinner = 2π radians, angular displacement of the spinner as one arm moves to the position of the 2nd arm:

$\theta = 1/3\ (2\pi)$ radians



Angular speed (ω) of a fidget spinner spinning at 62.5 Hz – convert to radians per second

$$62.5 \frac{rot}{s} * 2\pi \frac{radians}{rotation} = 393\, rad/sec$$

If centripetal force keeps an object in rotation, what force causes rotation or slows rotation down?

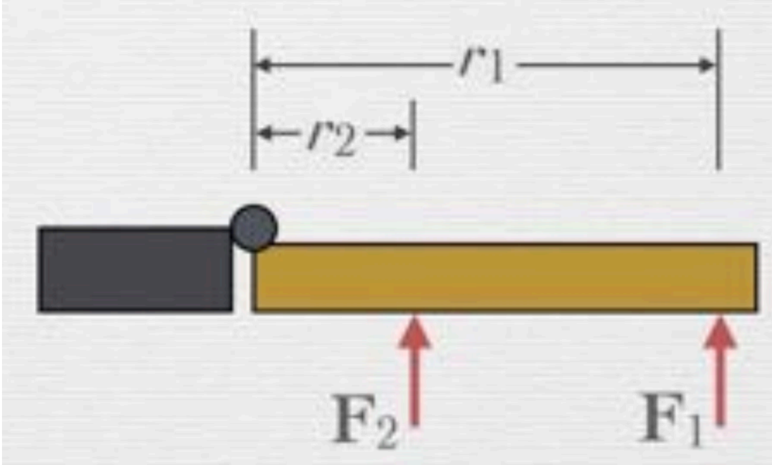
Torque = "Twisting force"

Torque = force times lever arm (or radius)

$$\tau = r \times F$$

Why hobbit doors are a stupid design

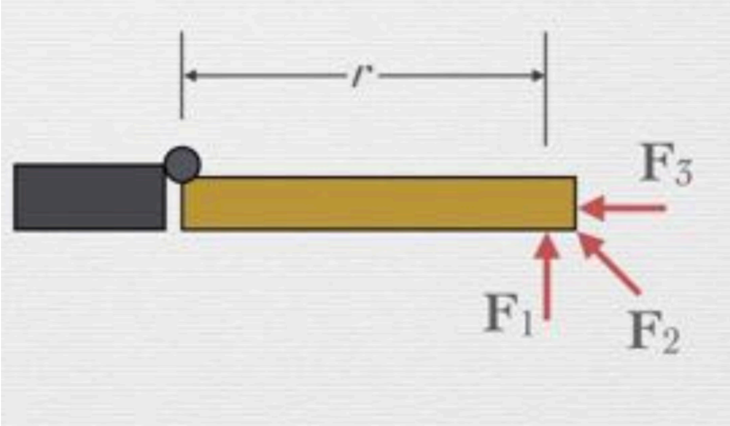
Where you apply a force to cause a torque matters
Apply a force closer to the hinge, the door doesn't open as quickly – smaller *r (lever arm)*, so smaller torque.

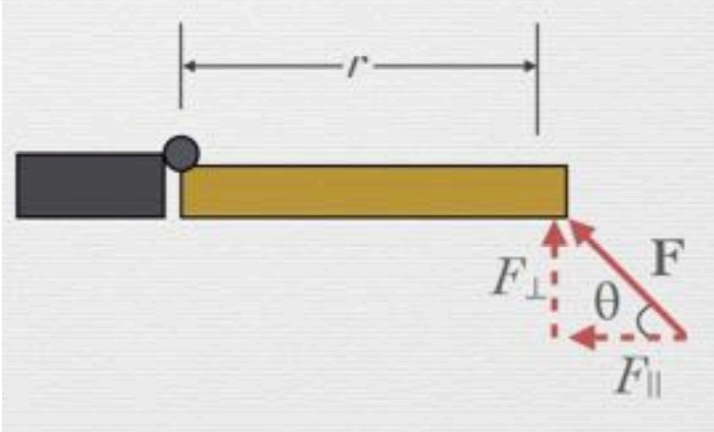


Only the *perpendicular* component of the force will contribute to rotation

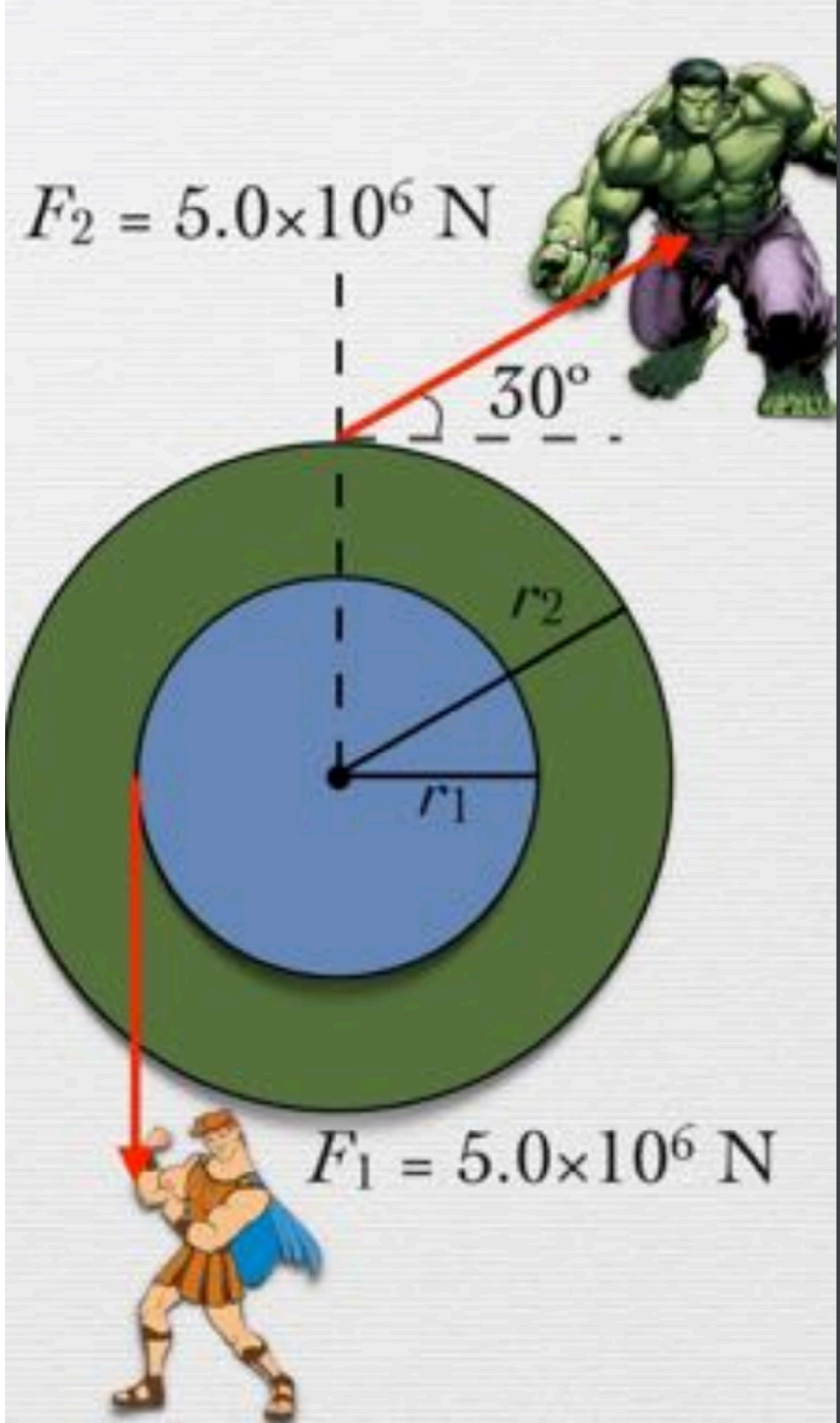
$$\tau = rF\sin\theta$$

θ (in degrees) is the angle between force applied and lever arm.





The Hulk and Hercules are in competition for some reason. They apply forces to 2 thin cylindrical wheels of radii $r_1 = 3.0$m and $r_2 = 5.0$ m that are attached to each other on an axel that passes through the center of each as shown. Who wins and what's the net torque?



$\boldsymbol{\tau}_{hulk} = (5.0 \text{ x } 10^6 \text{ N } \cos 30)(5\text{m}) = 2.2 \text{ x } 10^7 \text{ Nm}$

$\boldsymbol{\tau}_{herc} = (5.0 \text{ x } 10^6 \text{ N})(3\text{m}) = 1.5 \text{ x } 10^7 \text{ Nm}$

Net $\boldsymbol{\tau} = 2.2 \text{ x } 10^7 \text{ Nm} - 1.5 \text{ x } 10^7 \text{ Nm}$

Net $\boldsymbol{\tau} = 6.7 \text{ x } 10^6$ Nm clockwise : Hulk wins

Just like how Force causes linear acceleration,
Torque causes angular acceleration

$$\Sigma F = ma$$

$$\Sigma\tau = \mathrm{rma}$$

$$\Sigma\tau = \mathrm{rmr}\alpha$$

$$\Sigma\tau = (\Sigma mr^2)\alpha$$

$$\Sigma\tau = I\alpha$$

Rotational Inertia (or "Moment of Inertia") (*I*)

A measure of an object's "laziness" to changes in rotational motion

General equation :

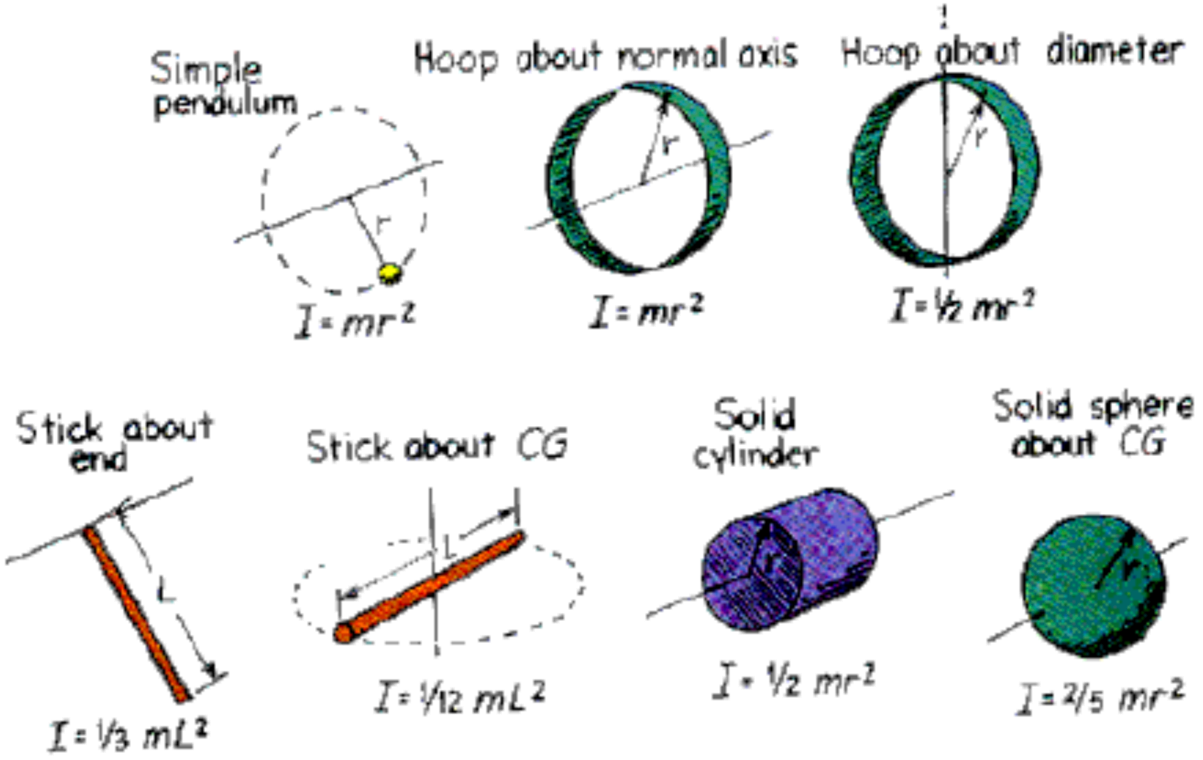
$$I = \Sigma m r^2$$

Depends on mass AND

distance of mass from axis of rotation

The farther away the mass is from the center, the more difficult it is to rotate an object

Rotational Inertia (or "Moment of Inertia") (I)



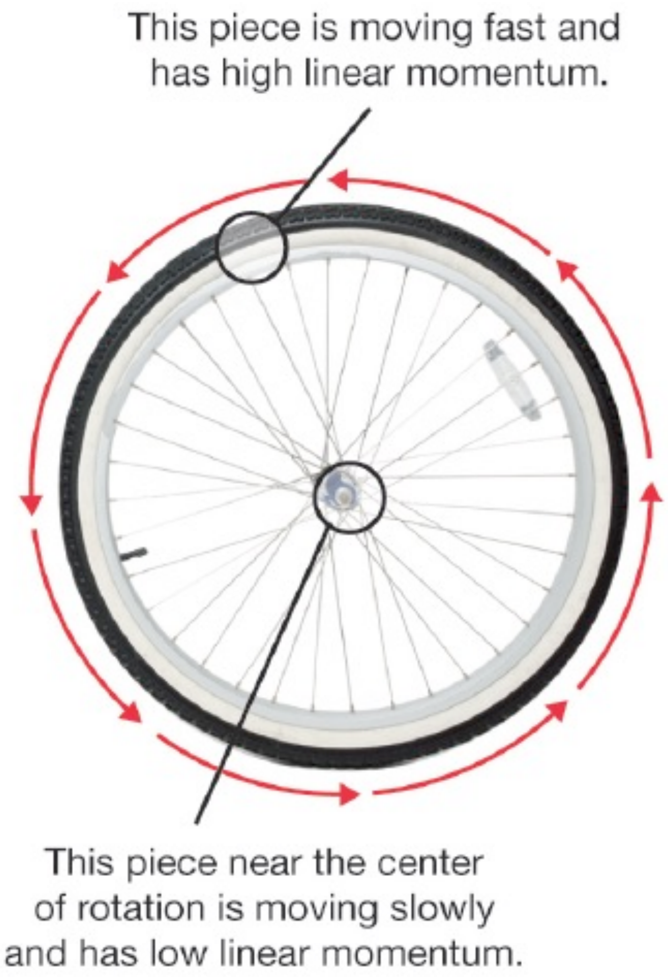
If you <u>need</u> a value for the rotational mass or the moment of inertia, you can always look it up. These are not things worth memorizing. However, you do need to know that and object with its mass far from the center has a greater moment of inertia than another object (of the same mass) with its mass near the center.

Balancing Pole increases Rotational Inertia

Angular Momentum

Momentum resulting from an object moving in linear motion is called ***linear momentum***.

Momentum resulting from the rotation (or spin) of an object is called ***angular momentum***.



Just like how Force causes a change in momentum,

$Ft = m\Delta v \rightarrow F = \Delta p/\Delta t$

(Newton's 2^{nd} Law: Momentum format)

$\boldsymbol{\tau}$ **causes** $\boldsymbol{\Delta}$**L** (change in angular momentum)

$\tau = \Delta L / \Delta t$

Calculating angular momentum

Angular momentum is calculated in a similar way to linear momentum, except the mass and velocity are replaced by the moment of inertia and angular velocity.

$$\vec{L} = I\,\vec{w}$$

Angular momentum (kg m/sec^2)

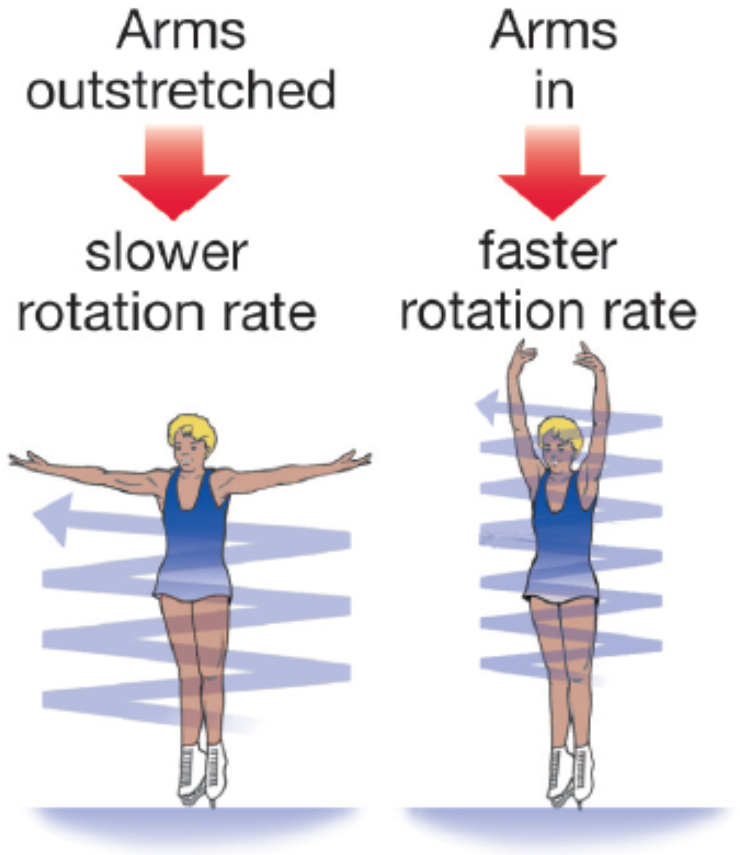
Moment of inertia (kg m^2)

Angular velocity (rad/sec)

Conservation of Angular Momentum

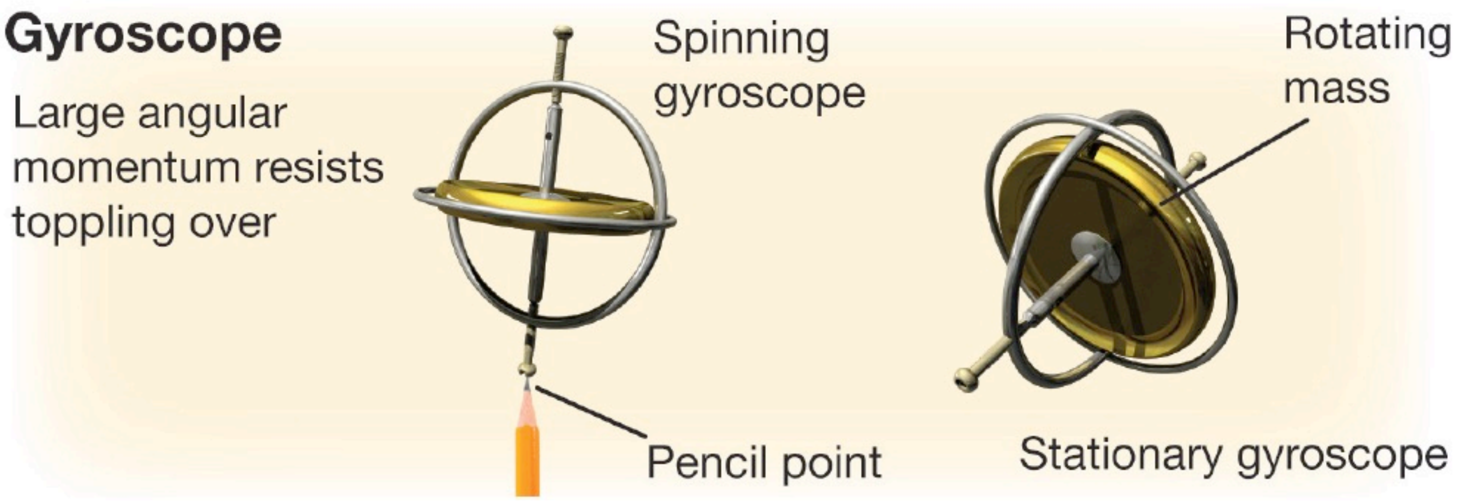
Angular momentum is important because it obeys a conservation law, as does linear momentum.

If there is no net torque acting on a system, angular momentum is conserved.

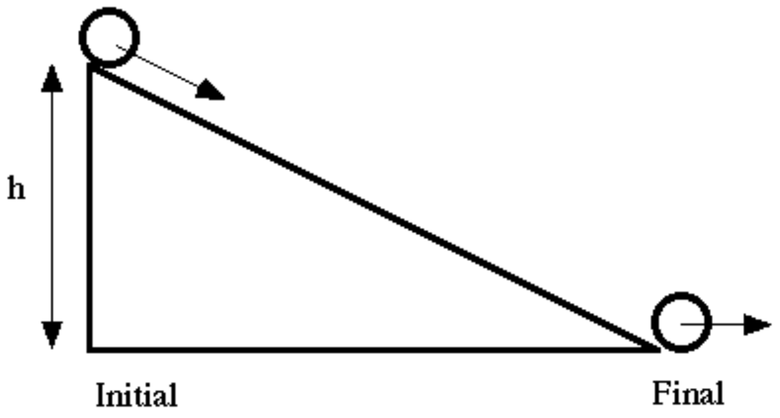


Gyroscopes

Fidget spinner can act like a gyroscope : Spinning fidget spinner is easier to balance on a pencil point than a spinner at rest.



Rotational KE



KE_{rot} = ½ I ω^2

Total KE = KE_{trans} + KE_{rot}

For any rolling object of mass *m* and radius *r*, what is its speed at the bottom of the hill?

PE_i = $Ke_{trans,\ final}$ + $KE_{rot,\ final}$

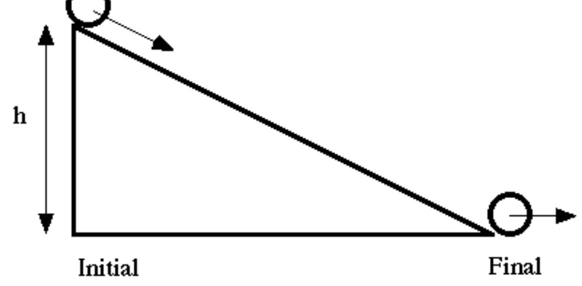
mgh = ½ mv^2 + ½ $I\omega^2$ I for any rolling object = c mr^2 $\omega = \frac{v}{r}$

mgh = ½ mv^2 + ½ (c mr^2)(v^2/r^2) C is some constant

gh = ½ v^2 + 2/10 v^2

v= $\sqrt{\frac{2gh}{1+c}}$ greater rotational inertia = smaller speed. Speed doesn't depend on mass or radius.

Rotational KE



Will a cylinder of soup or a cylinder filled with a more solid material reach the bottom first?

liquid: the insides slide more than they rotate, so less of the initial potential energy goes into rotational kinetic energy, and more can go into translational, moving the can down the incline.