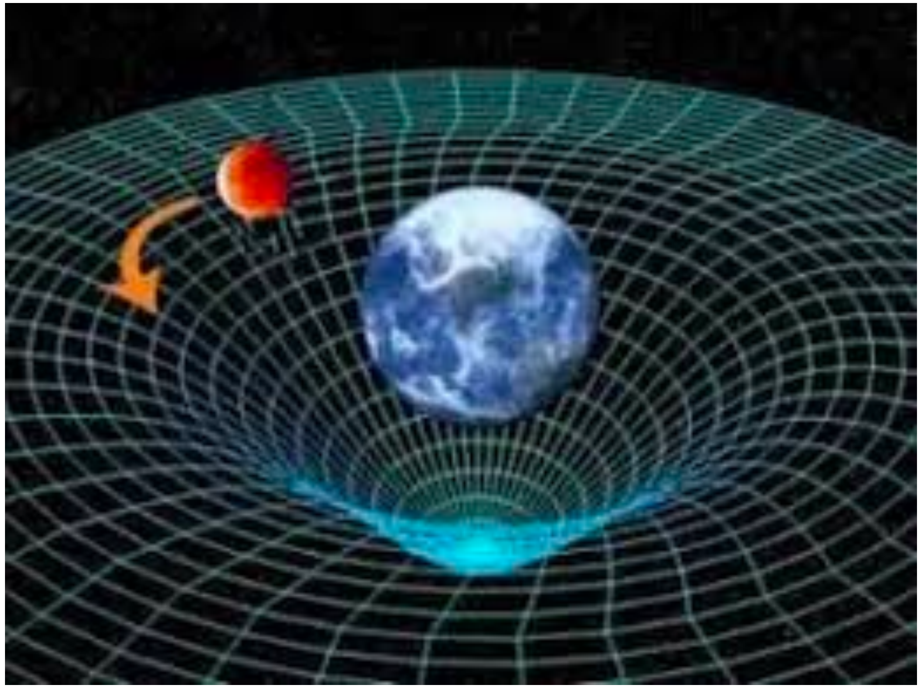
GRAVITY AND SPACE

Gravity

- Distortions of spacetime due to mass
- Mass doesn't weigh down spacetime, spacetime curves around a mass
- Attractive force



Force of Gravity between two objects

$$F_G = \frac{G M_1 m_2}{r^2}$$

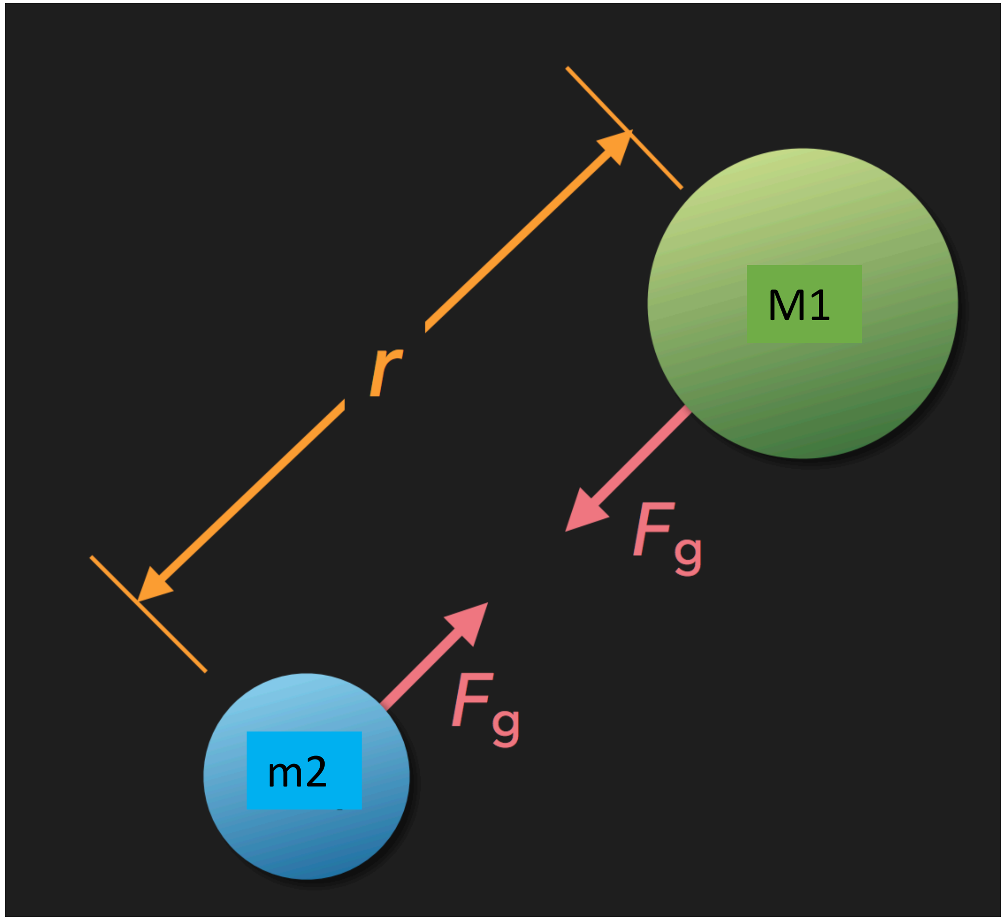
$G = 6.674 \times 10^{-11}\ Nm^2/kg^2$

Universal gravitation constant

r is distance between the **centers** of the objects

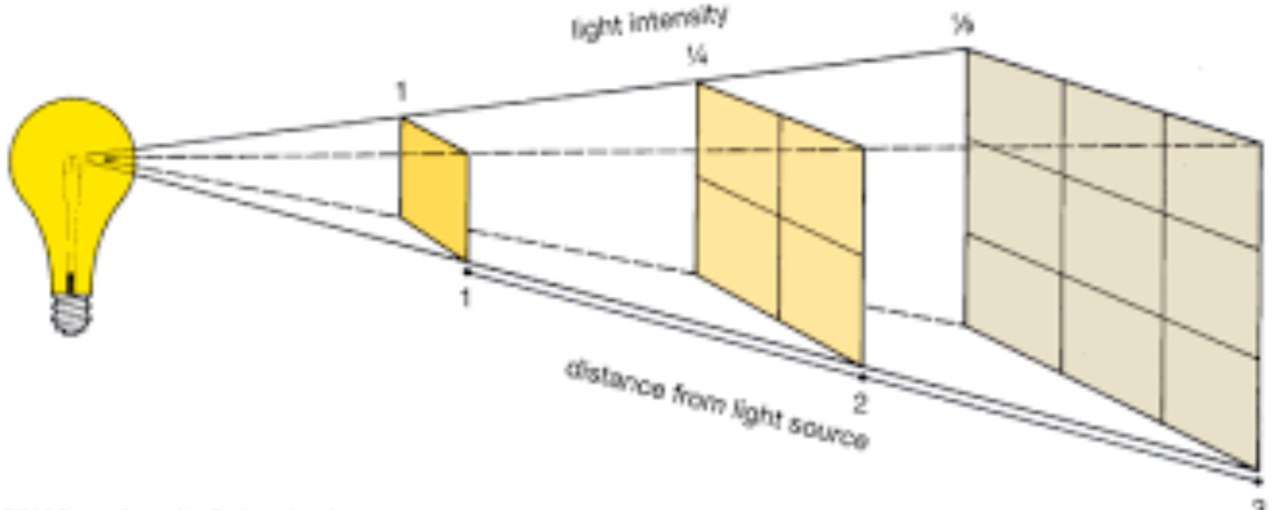
Which exerts a greater force:

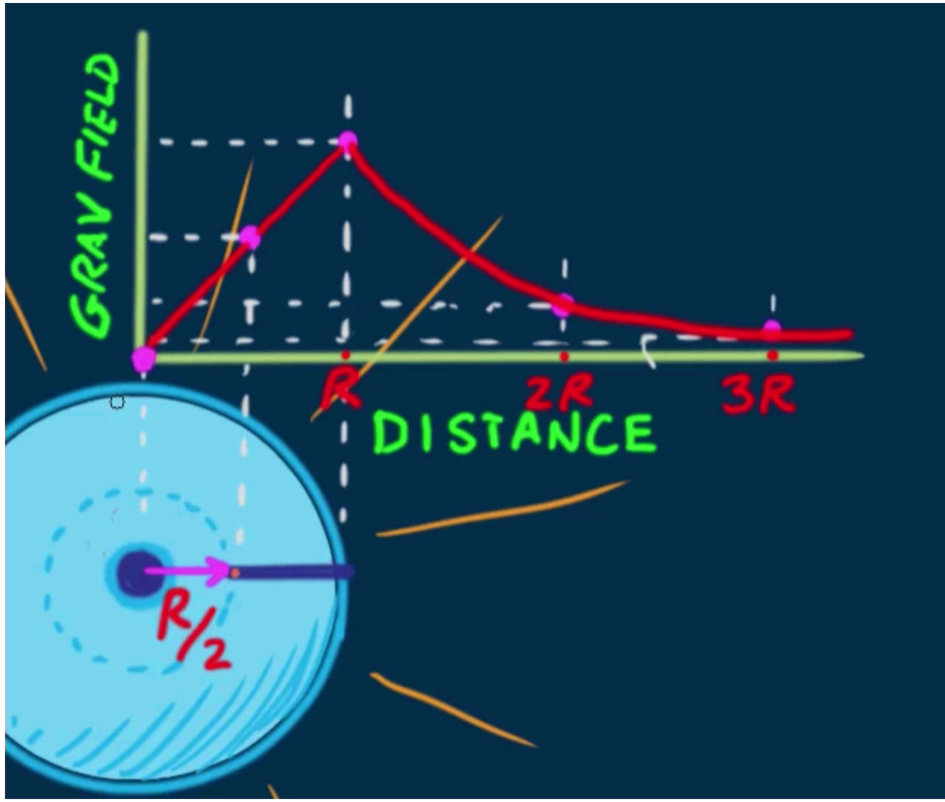
m1 on m2, or m2 on m1? (Hint: think of Newton's 3rd Law)



Inverse square law

- The force of gravity is proportional to the inverse square of the distance
  - If you double the distance, the force of gravity decreases by a factor of 4.





An apple on a tree feels 1 N of force due to gravity. If you double the height of the tree, the force of gravity on the apple would be:

A. 2x as strong
B. 1/2 as strong
C. ¼ as strong
D. None of the above

An apple on a tree feels 1 N of force due to gravity. If you double the height of the tree, the force of gravity on the apple would be:

A. 2x as strong
B. 1/2 as strong
C. ¼ as strong
D. <u>None of the above</u>

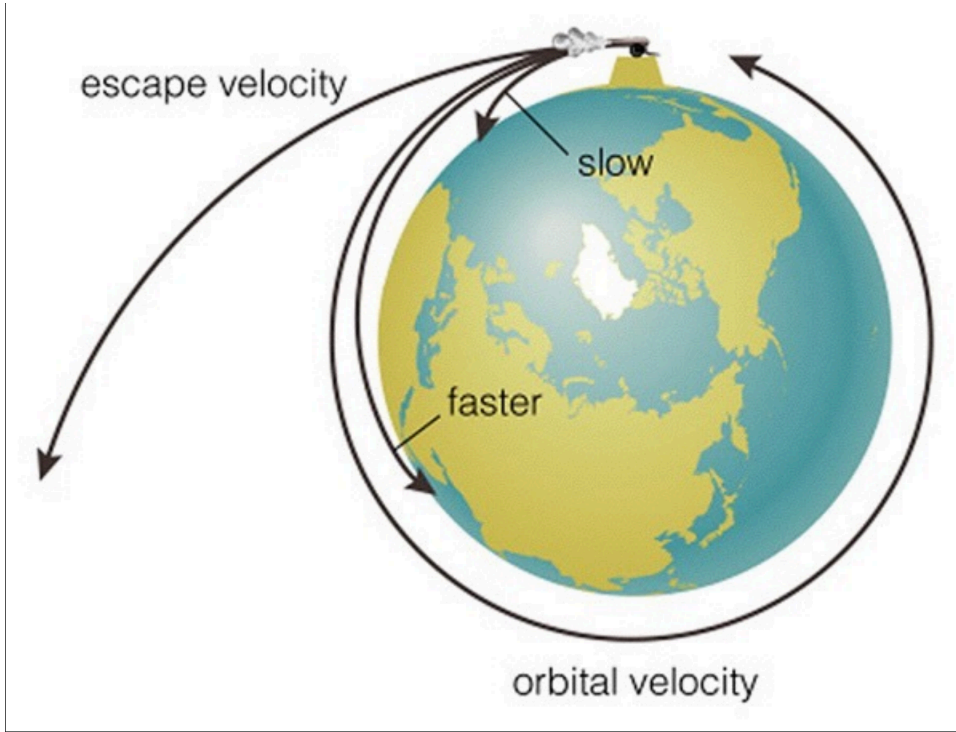
Why?

Gravitational Potential Energy → Escape speed

- $PE_G = \frac{-GM_1 m_2}{r}$
- How fast would you need to throw a ball of mass *m* from the surface of Earth for it to escape Earth's gravity?
- E_{tot} = KE + PE When *m* is no longer in Earth's gravity well, E_{tot} = 0
- $E_{tot} = \frac{1}{2}mv^2 - \frac{GM_E m}{r} = 0$
- $v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$ doesn't depend on the mass of the ball!

What is the escape speed of any object from the surface of Earth?

- $v_{esc} = \sqrt{\frac{2GM}{R}}$
- Mass of the Earth = 5.98 x 10^{24} kg, radius of Earth = 6.38 x10^6m
- 11,200 m/s
  - Or 11.2 km/s

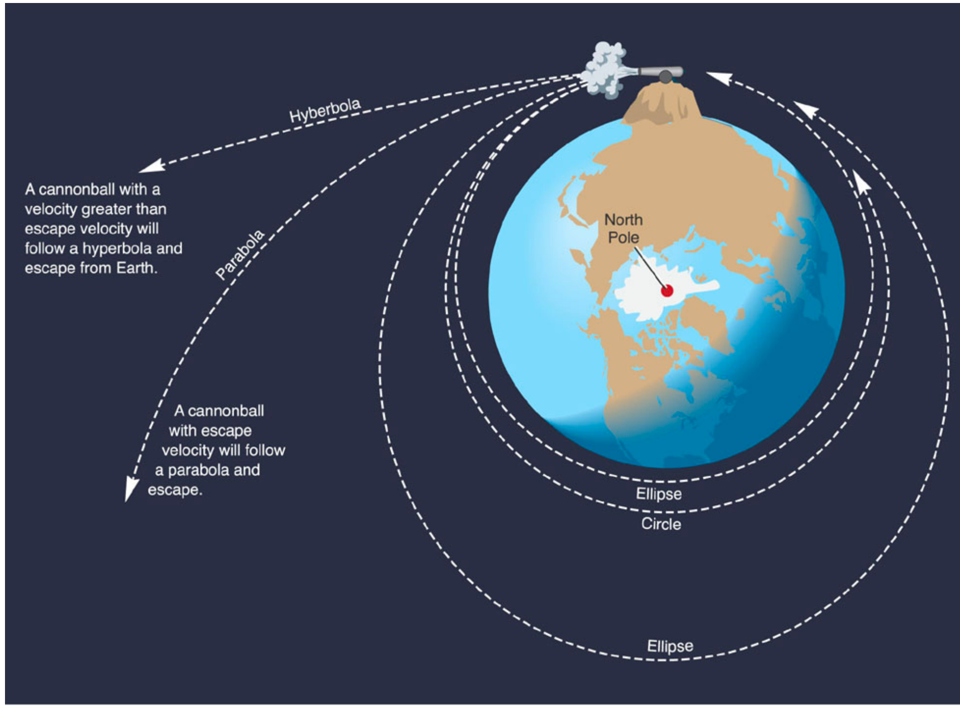


Which requires more fuel: a rocket going from Earth going to the Moon, or going from Moon to the Earth?

- Think escape velocity/which object has a larger gravity well

Orbiting speed

- If you threw an object horizontally at 8000 m/s, it would travel 8000 m horizontally and fall 5m. It would never hit the ground and orbit the earth in a circular orbit
- What if you threw it at 9000 m/s? This is not quite fast enough to escape Earth's gravity well (escape speed = 11200 m/s)
  - It would orbit in an ellipse!



"Weightlessness"

- Astronauts on ISS aren't really weightless
  - It's also a common misconception that there is no gravity in space
- They experience about 90% of the force of gravity from the Earth as we do
- They are constantly falling around the Earth

Orbiting speed

- $F_G = F_c$
- $\frac{GM_1 m_2}{r^2} = \frac{\mathrm{mv}^2}{\mathrm{r}}$

the *m* in the centripetal force equation is the one that is orbiting the earth, which gets cancelled out

- $v_{orb} = \sqrt{\frac{GM_1}{r}}$

Orbiting speed

- If an object is orbiting at height h from the surface of the Earth, what is its orbiting speed?
- $v_{orb} = \sqrt{\frac{GM_1}{r}}$
- r = R + h

R = radius of the Earth/object being orbited around

How does conservation of angular momentum explain why an object moves faster when it is closer to the sun vs. farther away from the sun?

- L = Iw

If an asteroid was very small but super massive, could you really live on it like the Little Prince?

If the asteroid was 1.75 m in radius, what mass would it need to have in order to have Earthlike gravity at its surface?

- g = 9.8 m/s^2
- $g = \frac{GM_1}{r^2}$
- M = 4.5 x10^{11} kg
  - That's just under 500 million tons, or the combined mass of every human on Earth…

What would be the escape speed on the surface?

- r = 1.75 m, M = 4.5 x10^{11} kg,

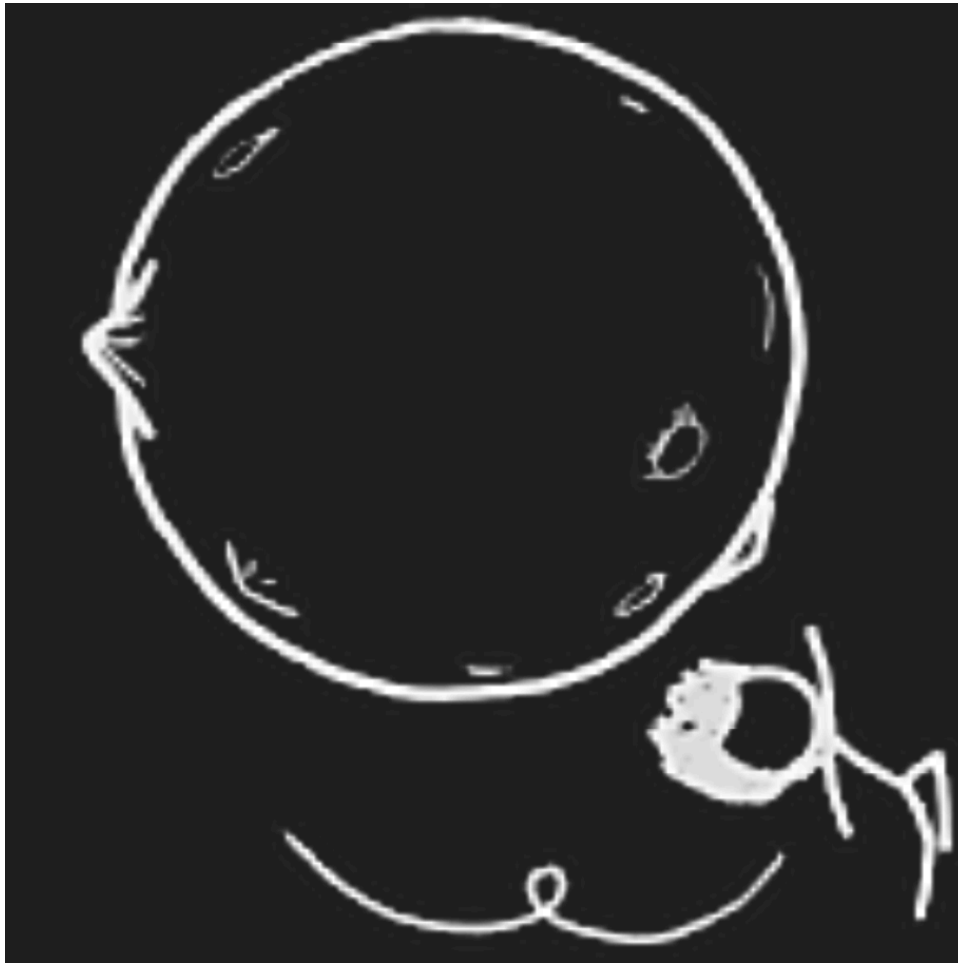
G = 6.674 x10^{-11} Nm^2/kg^2

- $v_{esc} = \sqrt{\frac{2GM}{R}}$ = 5.9 m/s
- If you can't dunk a basketball, you can't escape by jumping, but you might escape by running and jumping off a ramp

If your center of mass is 1.4 m above your feet, how fast would you need to run in order to orbit the asteroid?

- r = 1.75 m, M = 4.5 x10^{11} kg, G = 6.674 x10^{-11} Nm^2/kg^2

- $v_{orb} = \sqrt{\frac{GM}{R+h}}$ = 3.1 m/s
- That's a typical jogging speed



What would be the difference in the force of gravity at your head vs. your feet?

- Say you are 60 kg and 1.6 m (~5'3") tall
- $F_G = \frac{GM_1 m_2}{r^2}$
- $F_{G\ at\ head} = 160\ N$
- $F_{G\ at\ feet} = 588\ N$
- Called **tidal forces** – when there's a substantial difference in the force of gravity at different ends of a body.
- What happens with Earth's tides and what we think will happen to you if you got too close to a black hole
- It would feel like lying on a merry-go-round with your head near the center

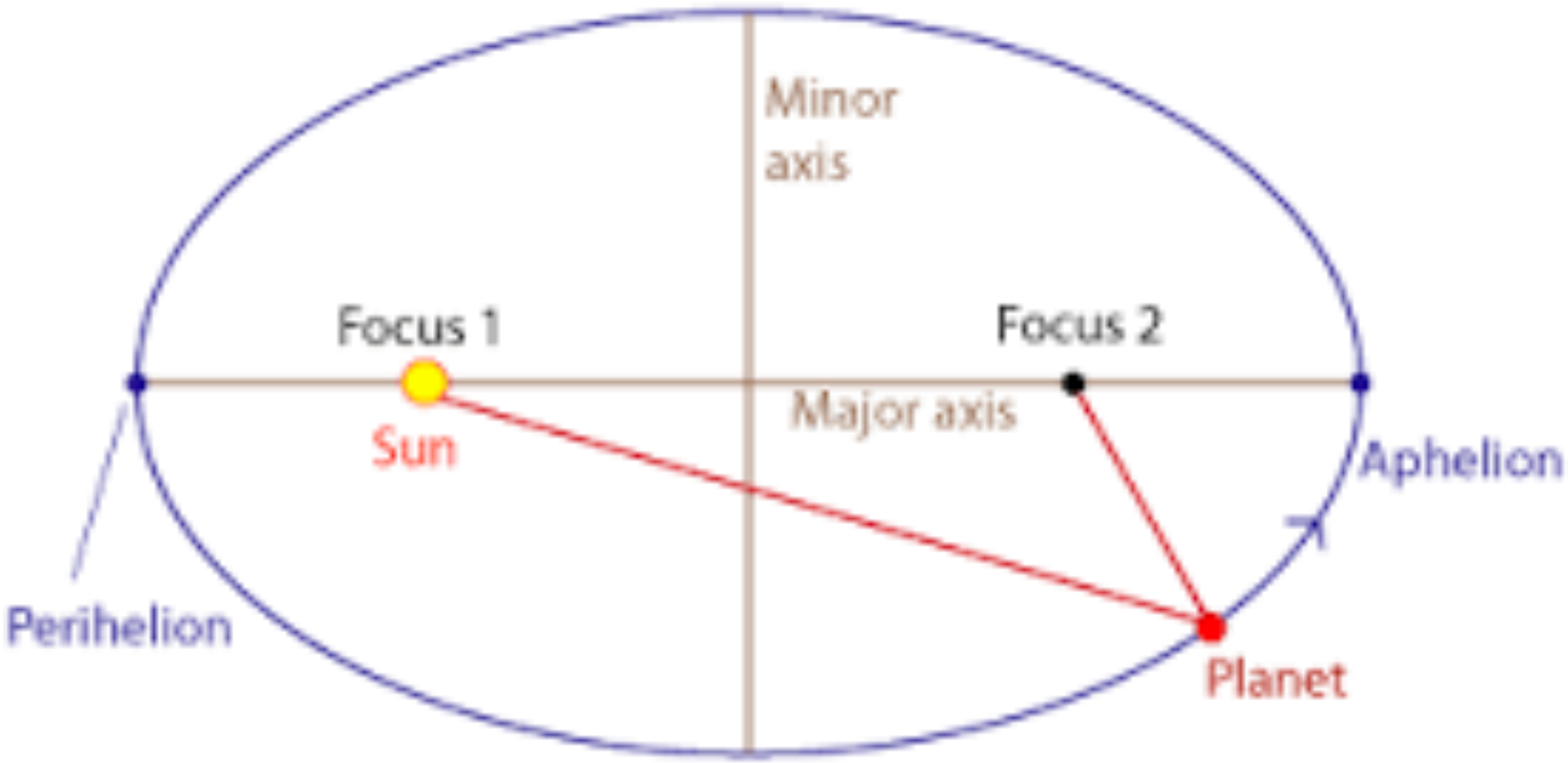
Kepler’s 3 Laws

- Johannes Kepler (1571-1630)
- German mathematician, astrologer, astronomer
- 3 Laws of Planetary Motion
  - 1) The orbit of a planet is an ellipse with the Sun at one of the two focus points 2) A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time 3) The square of the orbital period of a planet is proportional to the cube of its average distance from the Sun

Kepler’s 3 Laws

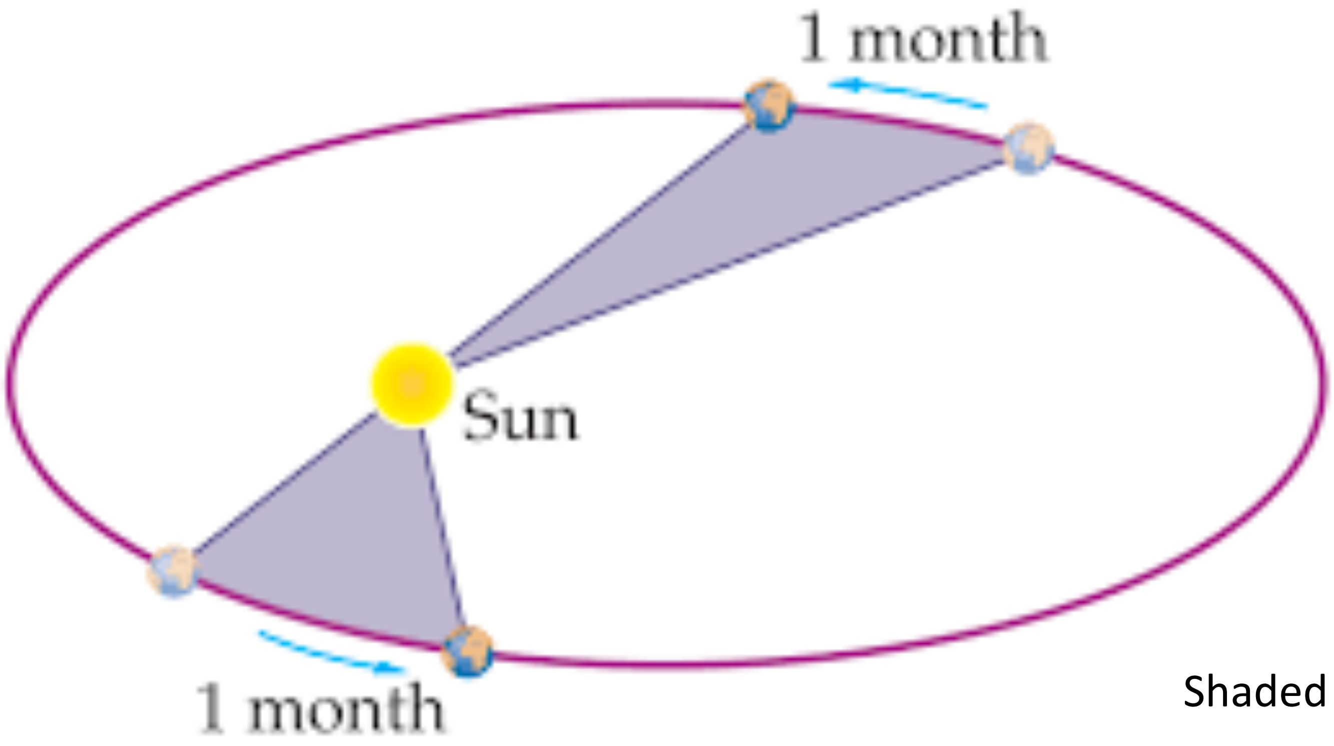
1) The orbit of a planet is an ellipse with the Sun at one of the two focus points
2) A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time
3) The square of the orbital period of a planet is proportional to the cube of its average distance from the Sun

Kepler's 1st Law



An elliptical orbit of a planet
(greatly exaggerated)

Kepler's 2^{nd} Law



(a)

Shaded areas are equal

Kepler's 3rd Law

$$v_{orb} = v_t$$

$$\sqrt{\frac{GM_1}{r}} = \frac{2\pi r}{T}$$

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$T^2 = r^3$$

If expressed in the following units:

T	Earth years
r	Astronomical units AU (a = 1 AU for Earth)

T is in seconds and r is in meters
r is the distance between the centers of the two objects, not the radius of the object!

Use Kepler's 3^{rd} Law to find the mass of the Sun, if Earth's period is 1 year = 3.154×10^{7} seconds and the distance between the Earth and the sun is 1.5×10^{11} m

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Solve for M:

2.0×10^{30} kg