2D (OR 3D) ELASTIC COLLISIONS

- Vector nature of momentum is important
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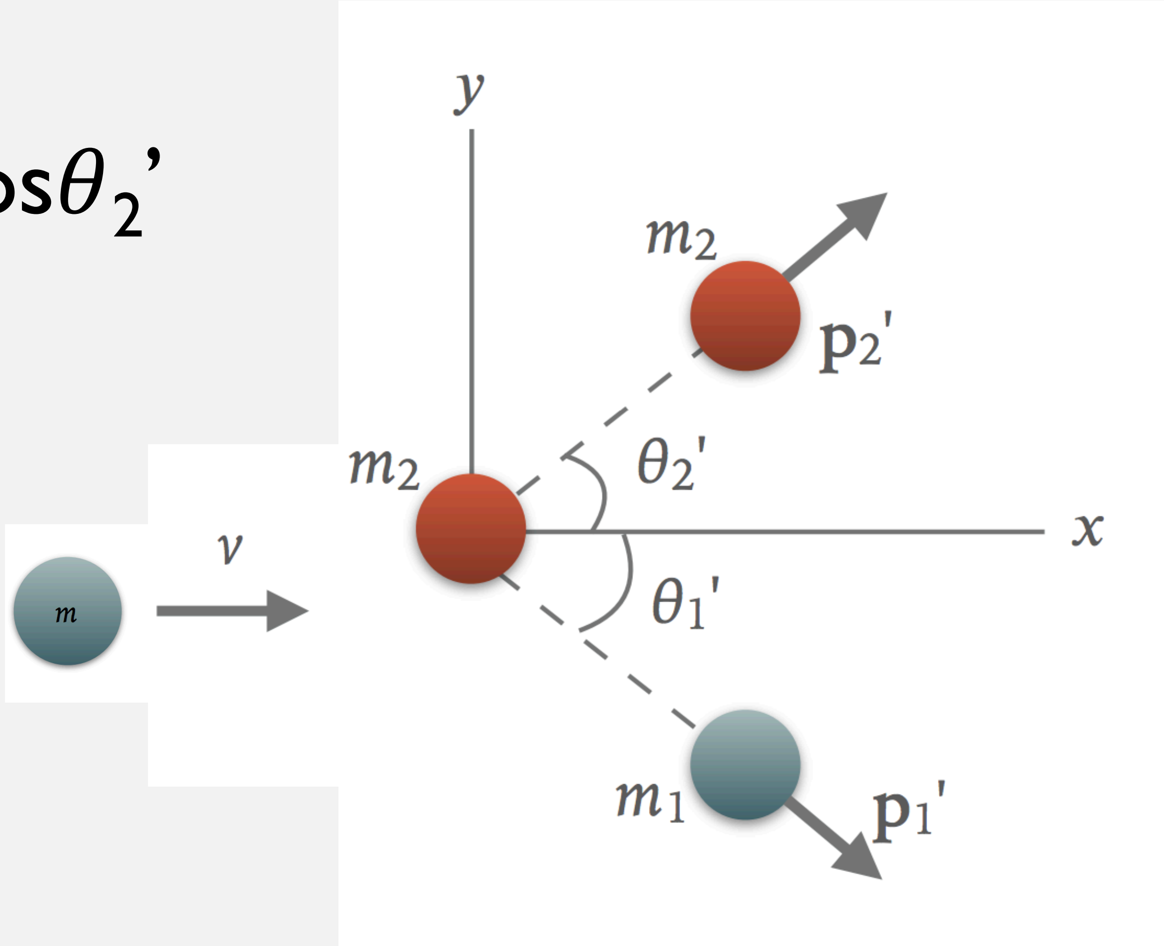
- $p_{1x} + p_{2x} = p_{1x}' + p_{2x}'$
  - $m_1 v_1 = m_1 v_1' \cos\theta_1' + m_2 v_2' \cos\theta_2'$
- $p_{1y} + p_{2y} = p_{1y}' + p_{2y}'$
  - $0 = m_1 v_1' \sin\theta_1' + m_2 v_2' \sin\theta_2'$



2D OR 3D COLLISIONS

- A billiard ball moving with speed v_1 = 3.0 m/s in the +x direction strikes an equal-mass ball initially at rest
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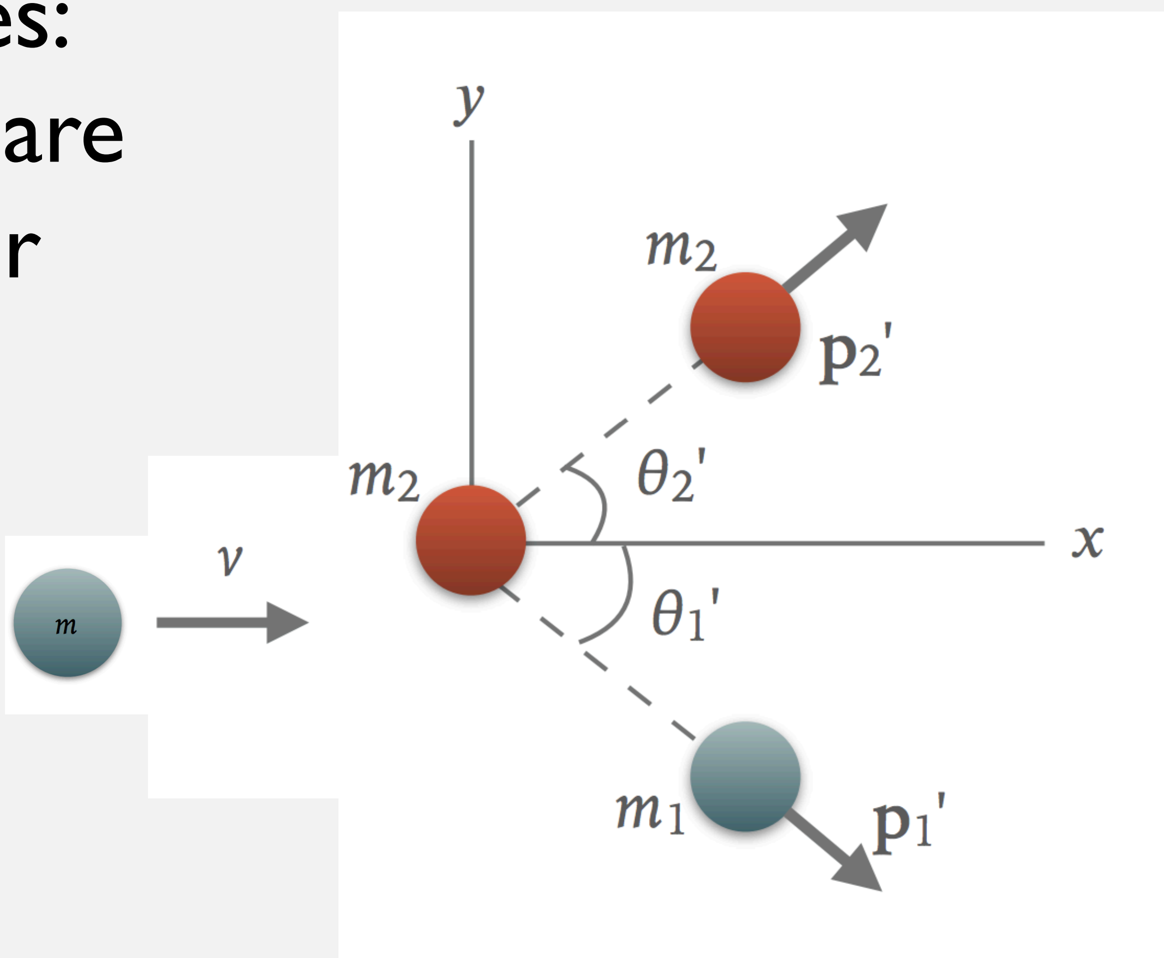
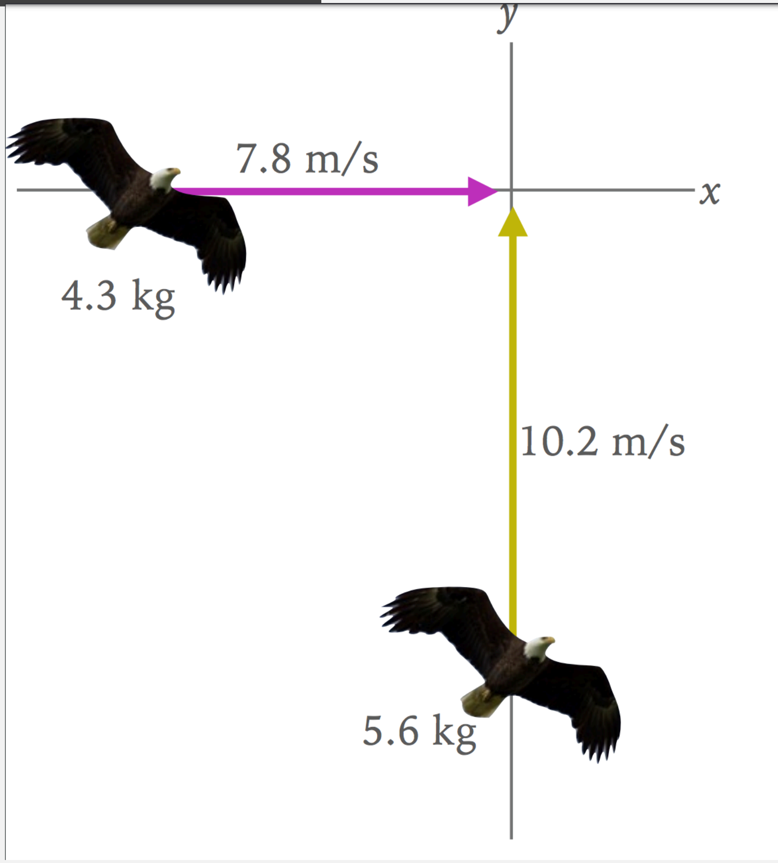
- Both 2.1 m/s



2D OR 3D INELASTIC COLLISIONS

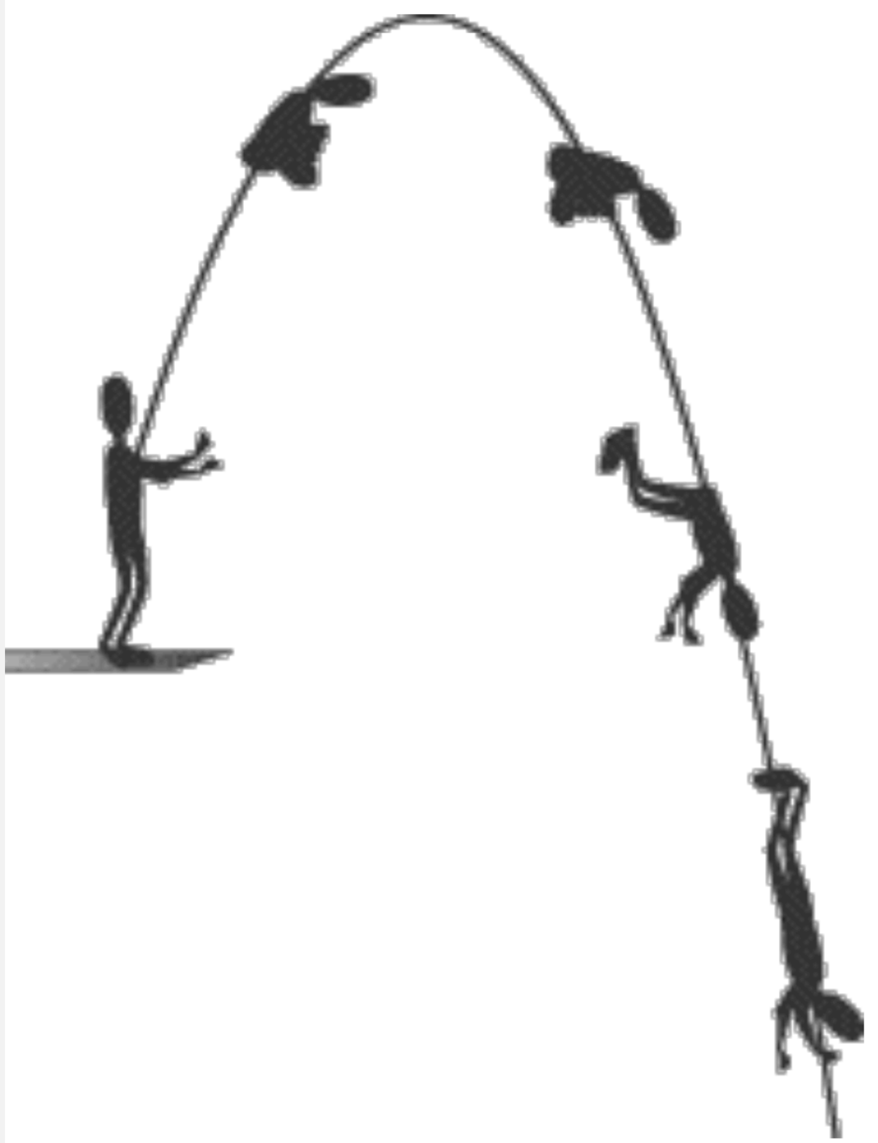
- An eagle (m_1 = 4.3 kg) moving with speed v_1 = 7.8 m/s is on a collision course with a second eagle (m_2 = 5.6 kg) moving at v_2 = 10.2 m/s in a direction at right angles to each other. After they collide they hold onto each other.
- In what direction and with what speed are they moving after the collision?
- v' = 6.7 m/s at 60 degrees



- Why do high jumpers jump backwards and arch their backs to get over the bar?
- How do high jumpers use the concept of the center of mass to jump higher with the same amount of force?

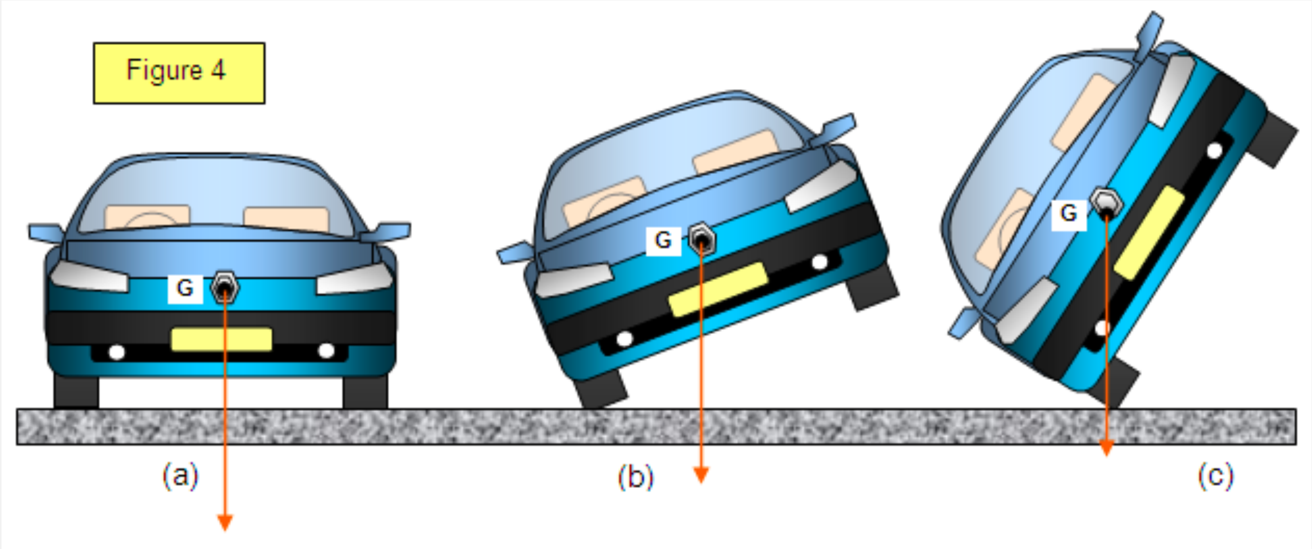
https://www.youtube.com/watch?v=RaGUW1d0w8g

- In real life, objects aren't just points
- Real, extended bodies can undergo rotation, vibration, etc. in addition to translational motion
- The diver experiences parabolic translational motion and rotational motion
- Motion that is not pure translational = general motion

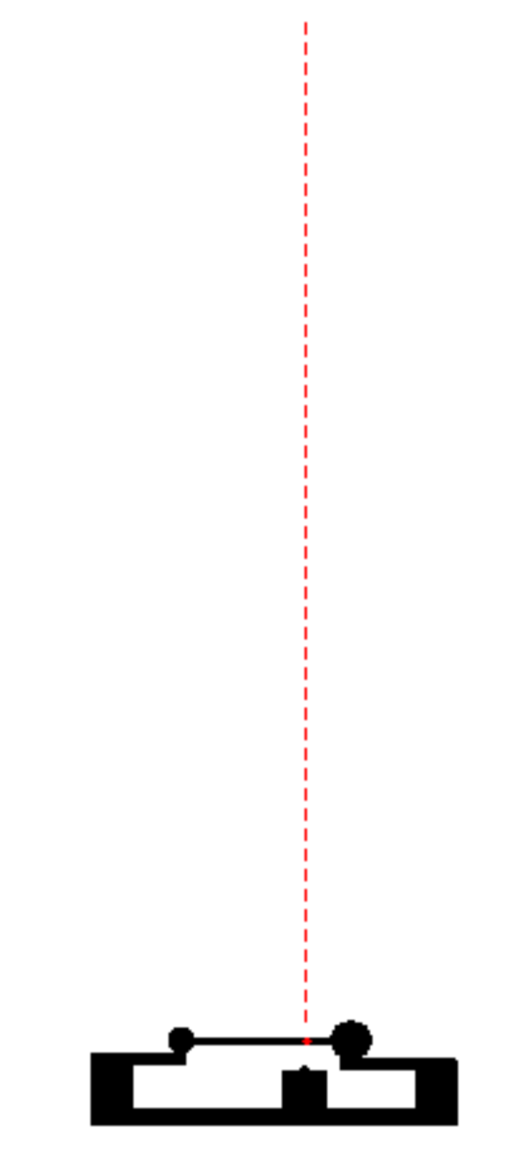


CENTER OF MASS

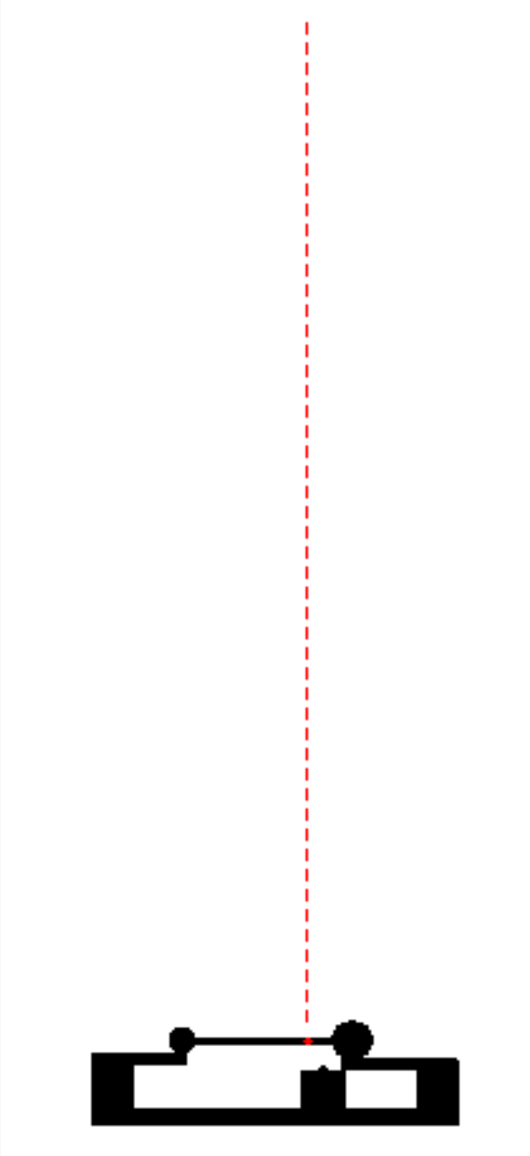
- Center of mass (CM) is the point where:
  - All mass is considered "concentrated"
  - Net force can be applied without causing object to rotate
  - Object can be balanced



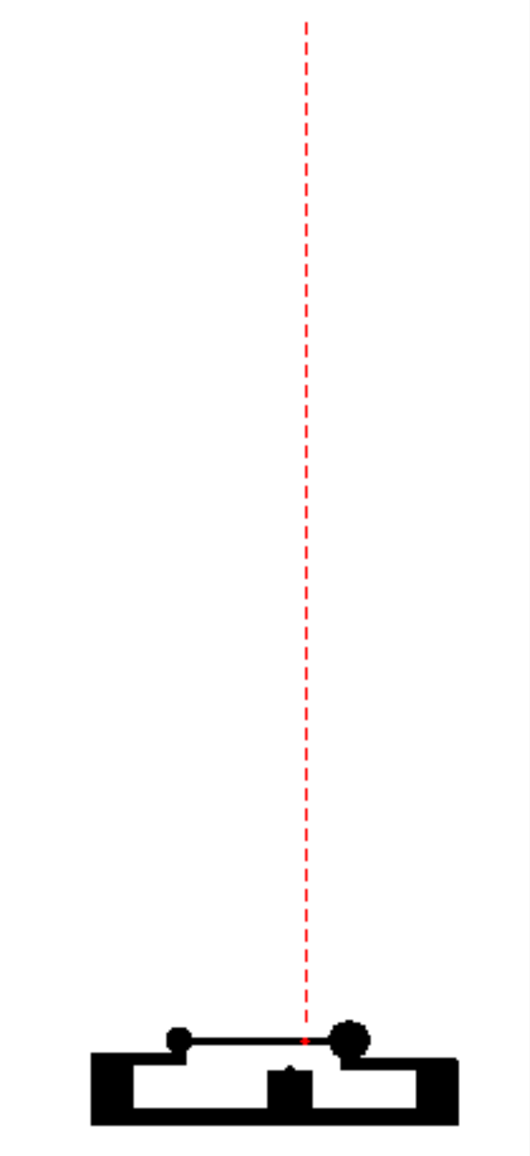
F on CM



F right of CM



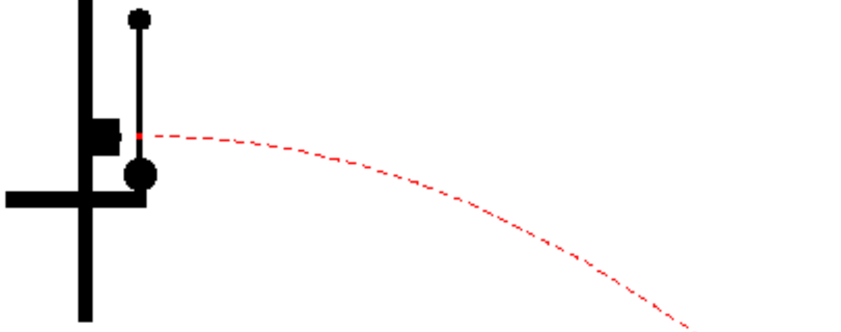
F left of CM



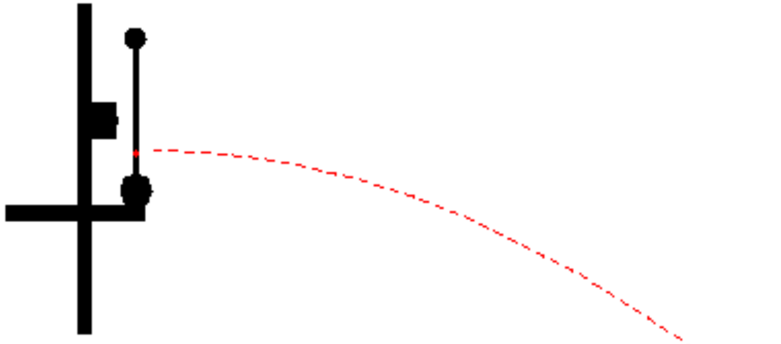
http://www.acs.psu.edu/drussell/Demos/COM/com-a.html

CENTER OF MASS: 2D MOTION

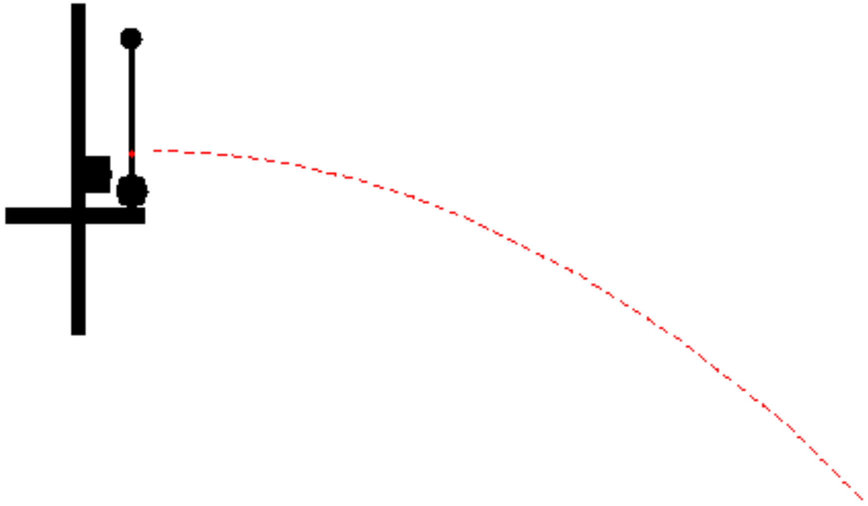
F on CM



F above CM



F below CM



Apply a force away from the center of mass, object will rotate around its center of mass

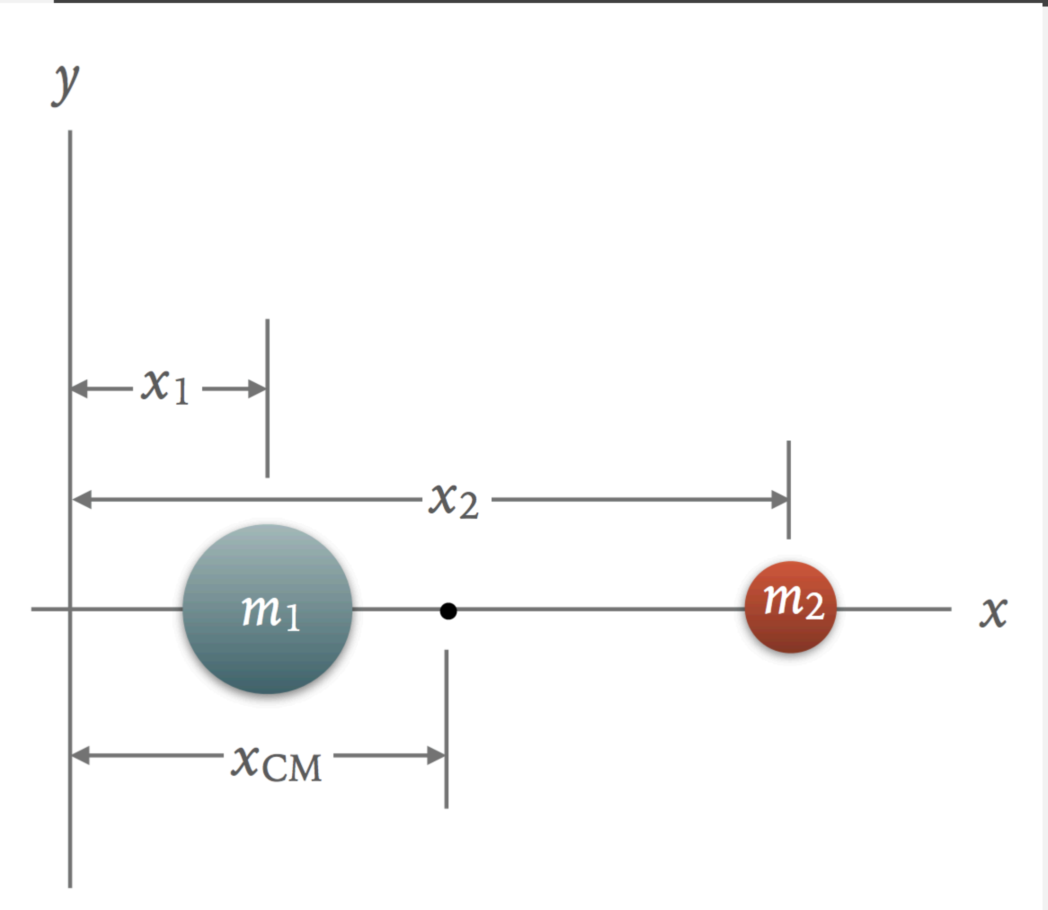
http://www.acs.psu.edu/drussell/Demos/COM/com-a.html

CENTER OF MASS (CM)

CM doesn't need to be inside the object in question!

CM is often found experimentally or mathematically

CENTER OF MASS (CM)

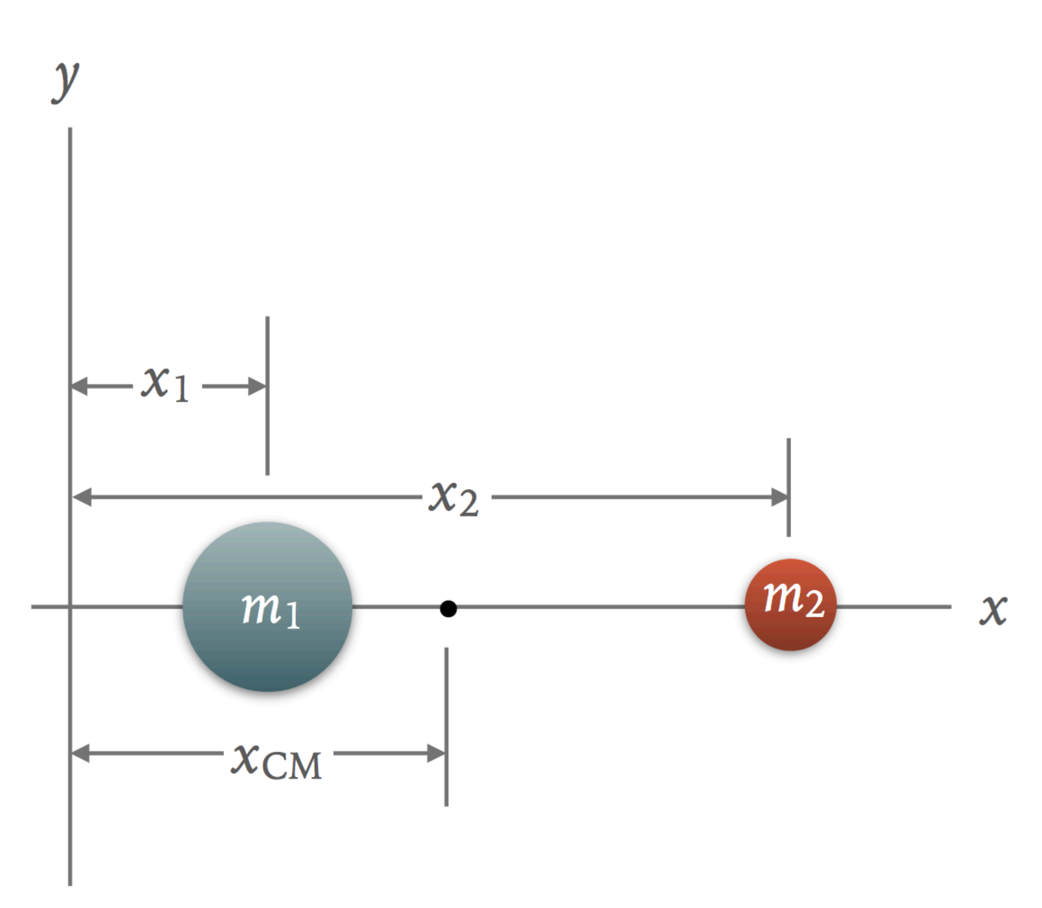


Any extended body is made up of many tiny particles

For a 2-particle system, the position of the center of mass (x_{CM}) is:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

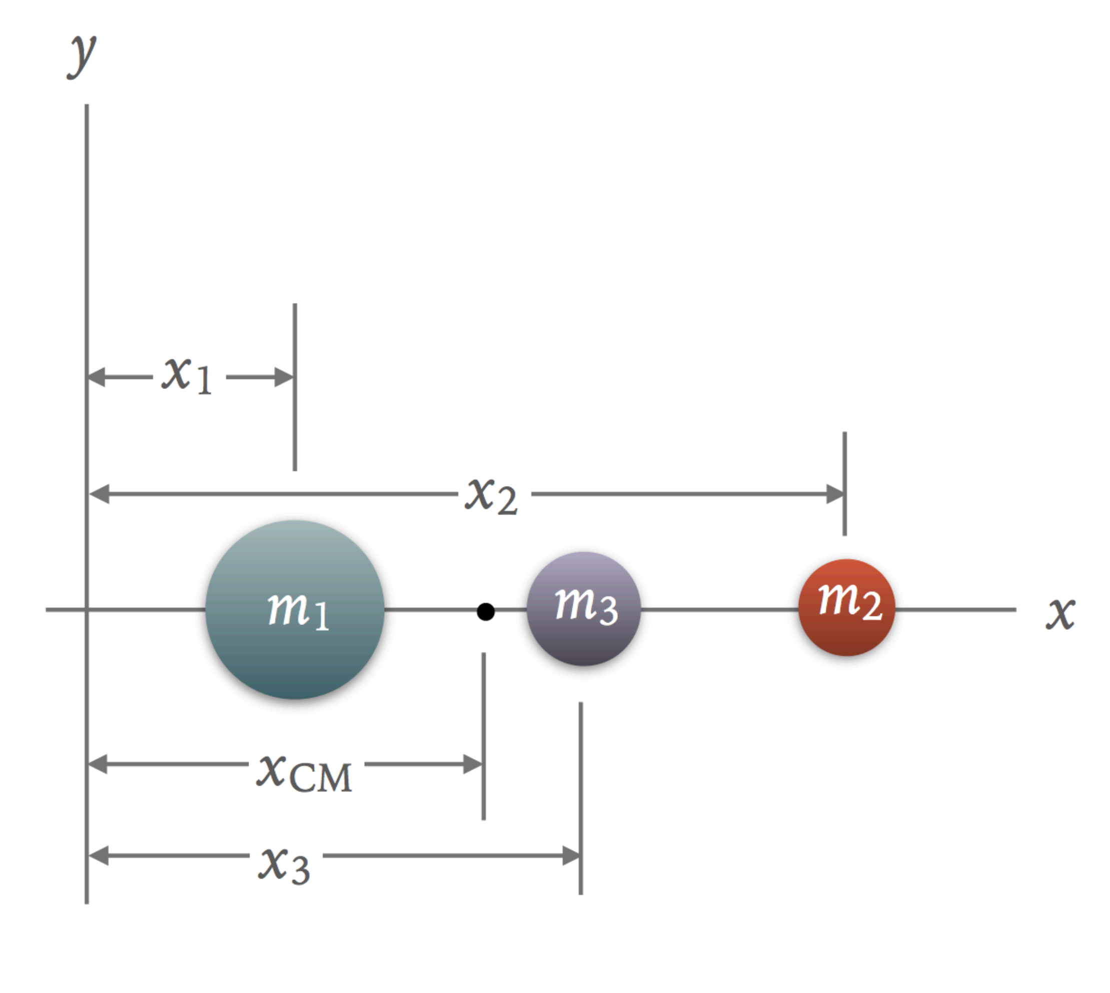
CENTER OF MASS (CM)



Which of the 2 masses is x_{CM} closer to? Why?

Where would you find x_{CM} if the masses were equal?

CENTER OF MASS (CM)



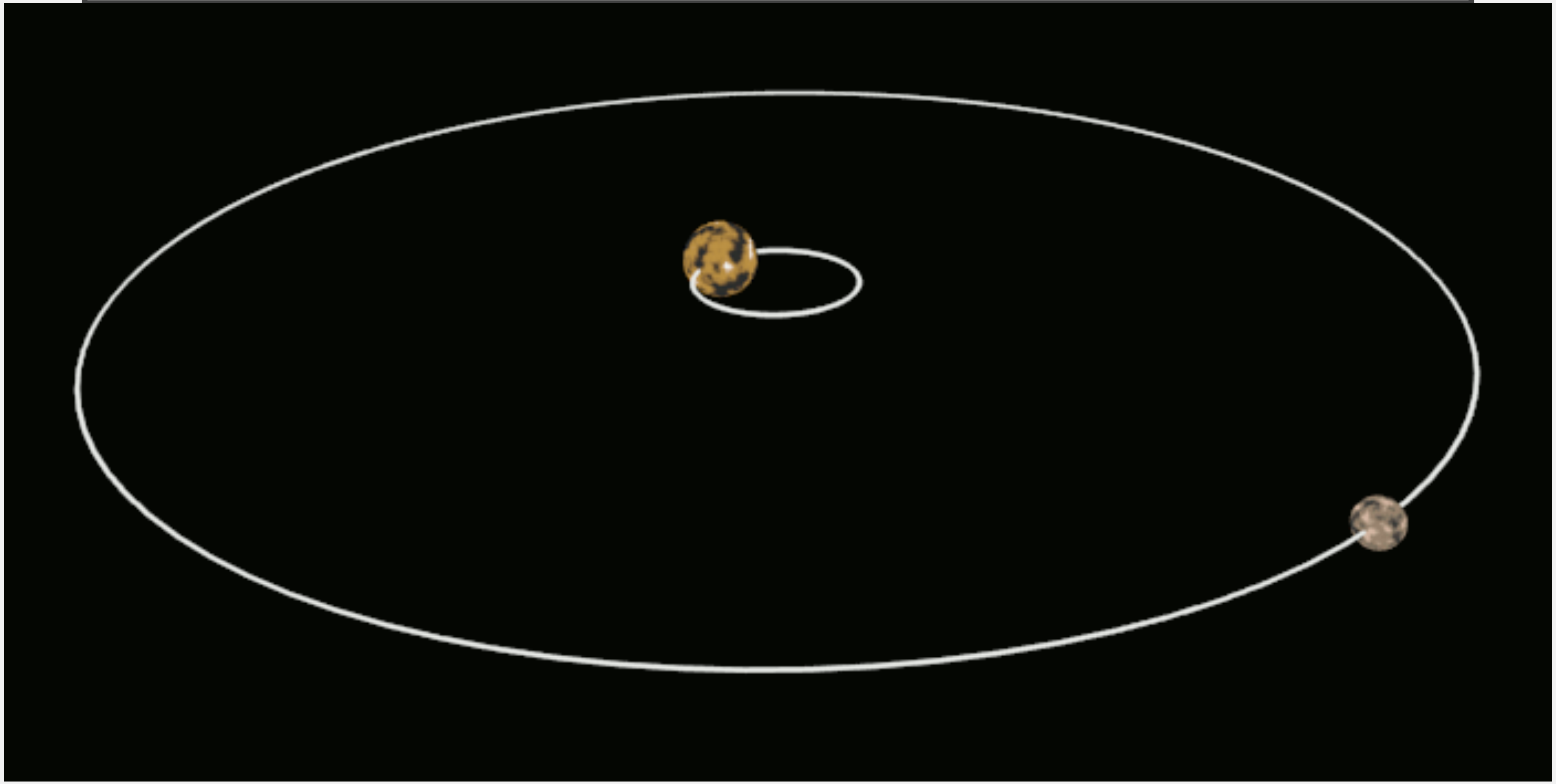
Let's add a 3rd mass.

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

We could do this forever!

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

CENTER OF MASS (CM): PLUTO AND CHARON



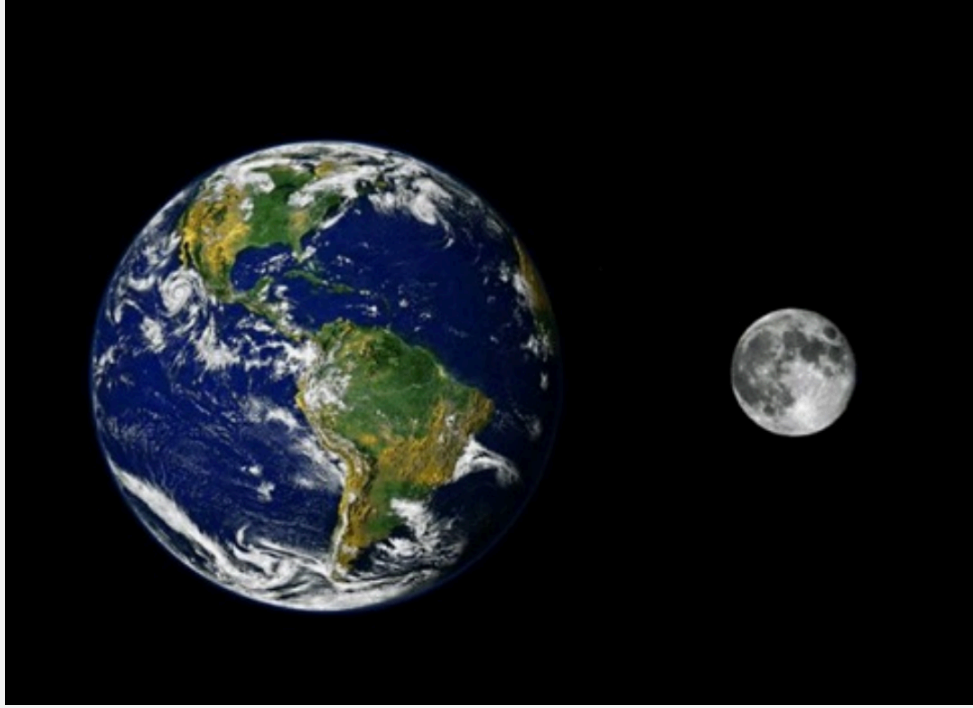
CALCULATING CM

The masses of the Earth and Moon are 5.98×10^{24} kg and 7.35×10^{22} kg, respectively, and their centers are separated by 3.84×10^{8} m

Where is their center of mass?

4.66×10^{6} m from the center of the Earth

This is within the radius of the Earth, which is about 6.4×10^{6} m

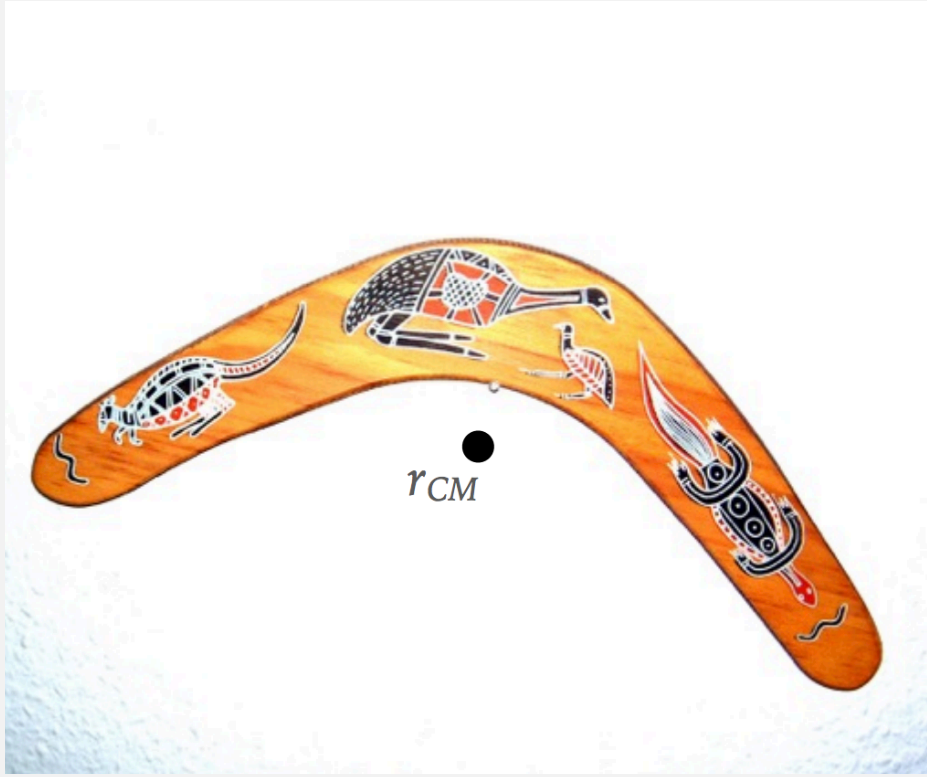


$$x_{cm} = \frac{x_E m_E + x_M m_M}{m_E + m_M}$$

For objects in 2 or 3 dimensions, you would need to find the center of mass in the x, y, and z directions

$$y_{com} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$$

$$z_{com} = \frac{\sum_{i=1}^{n} m_i z_i}{\sum_{i=1}^{n} m_i}$$



MOTION OF CM

Same calculations apply to finding the velocity/acceleration of a system's center of mass

$$v_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum_{i=1}^{n} m_i}$$

$$a_{cm} = \frac{\sum_{i=1}^{n} m_i a_i}{\sum_{i=1}^{n} m_i}$$

$$x\ (or\ y)_{cm} = \frac{\sum_{i=1}^{n} m_i x(or\ y)_i}{\sum_{i=1}^{n} m_i}$$

$$v_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum_{i=1}^{n} m_i}$$

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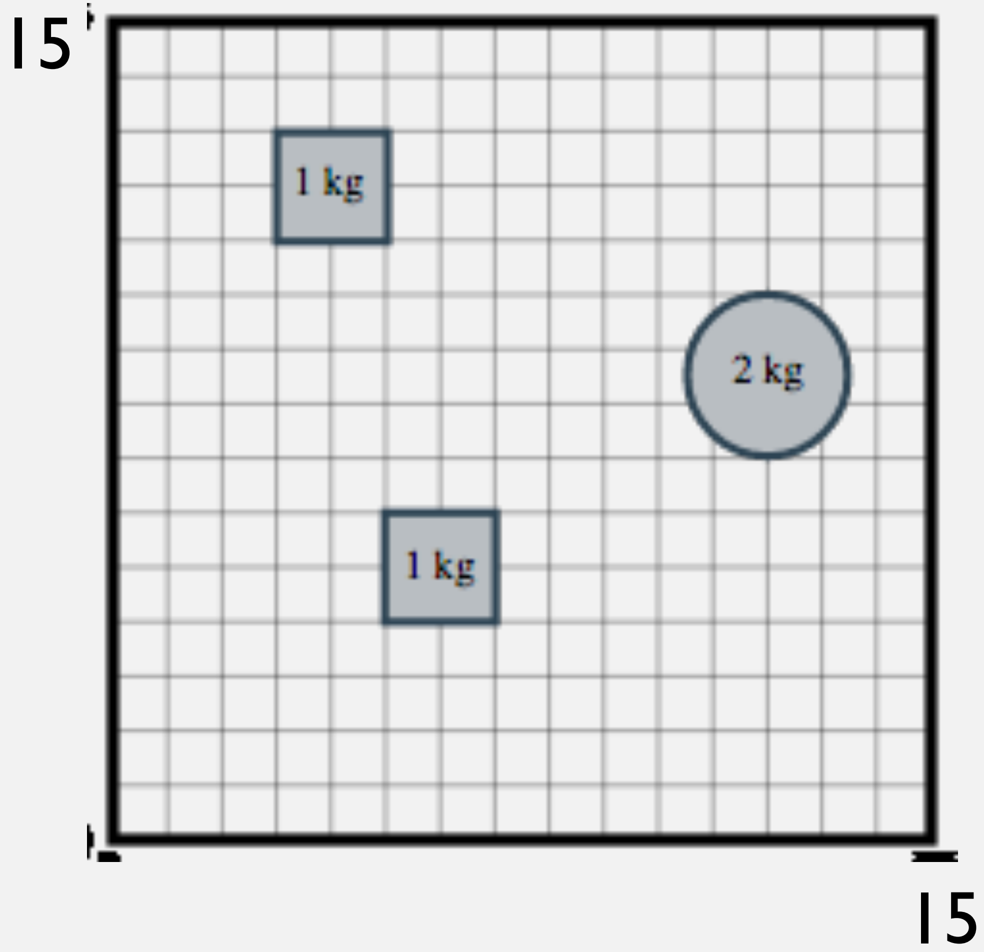

CENTER OF GRAVITY (CG)

Center of Gravity (CG) is the point at which the force of gravity can be considered to act

Usually the same point as CM

- When would CM not be the same as CG?

If an object is so large that the gravitational field around it isn't uniform…

- https://phet.colorado.edu/sims/html/balancing-act/latest/balancing-act_en.html
- Go to the Game section, and start from level 4 ☺
- $x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$

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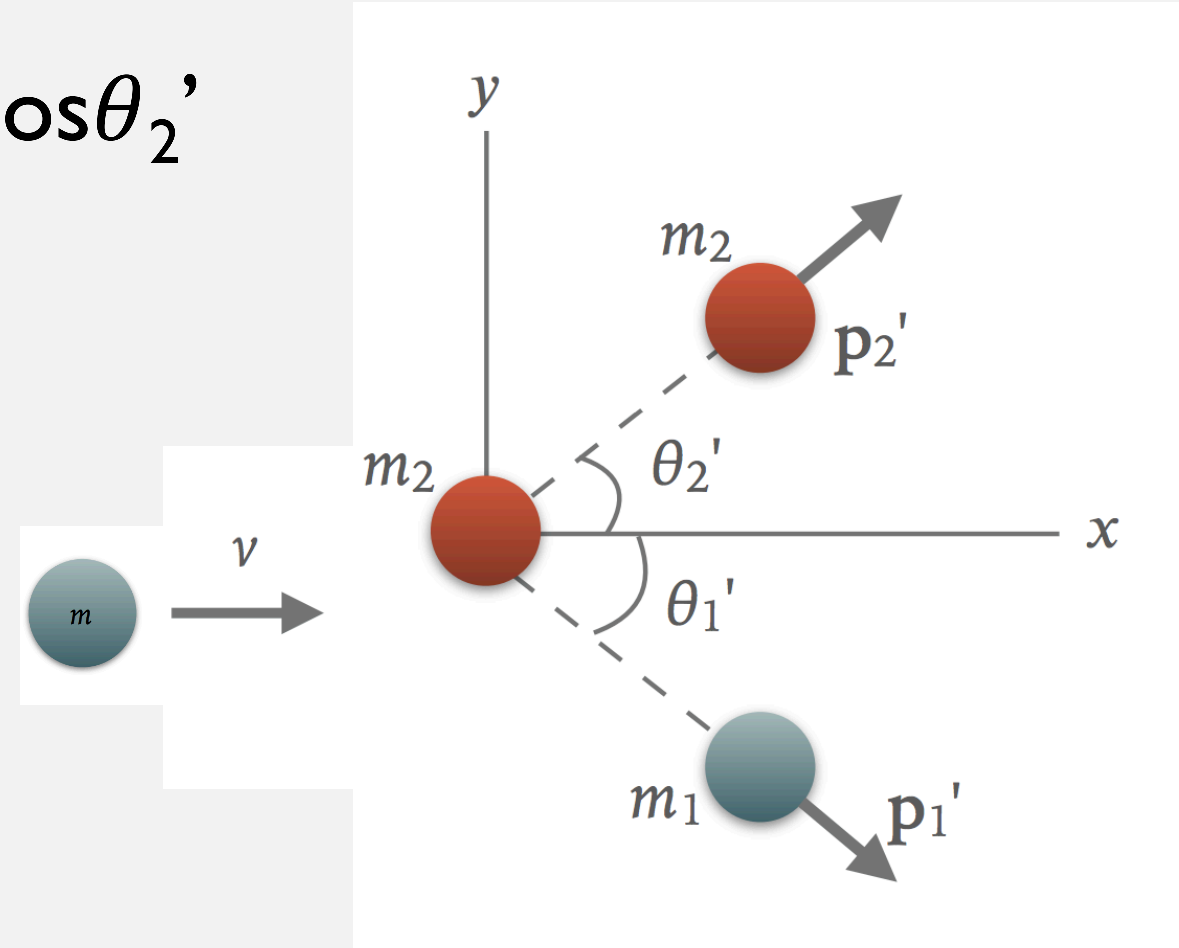

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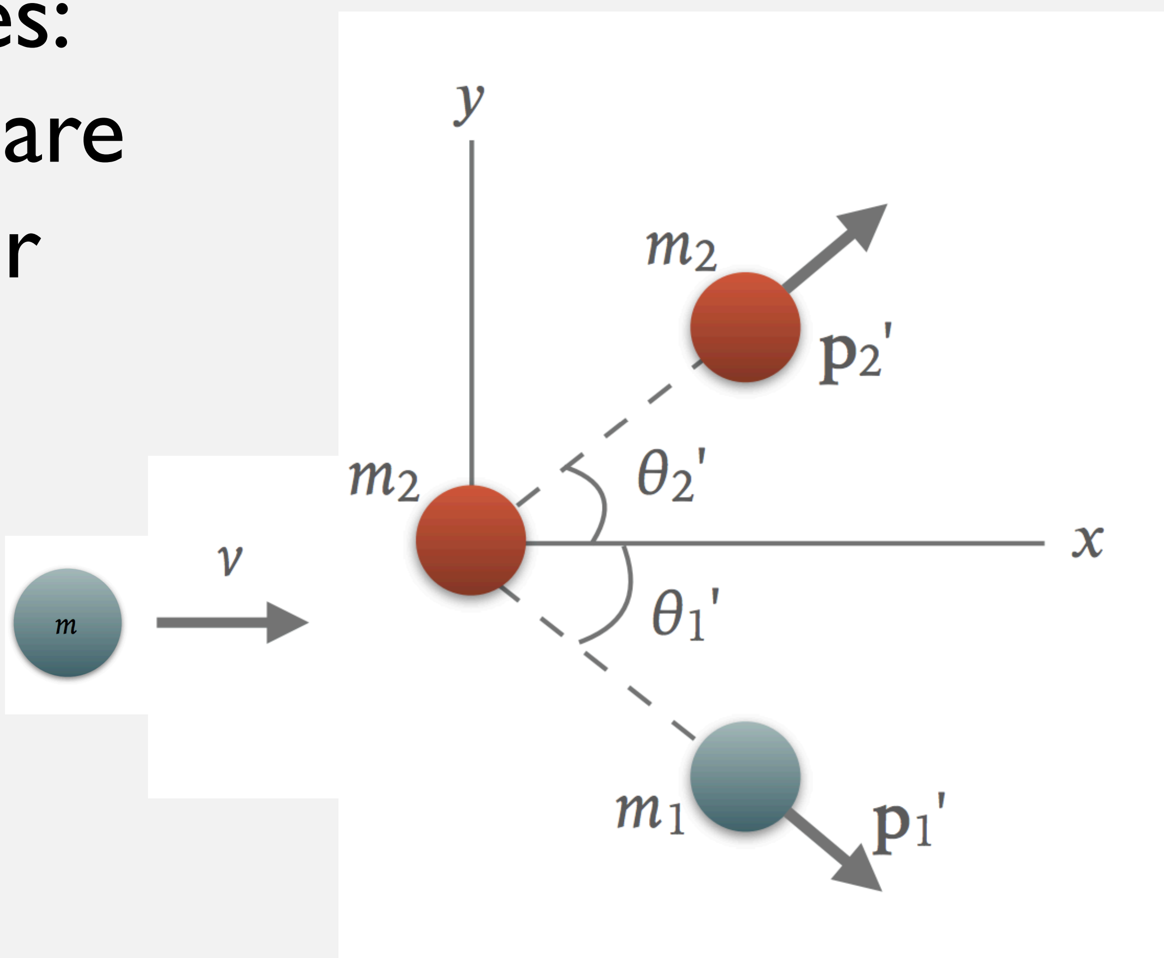
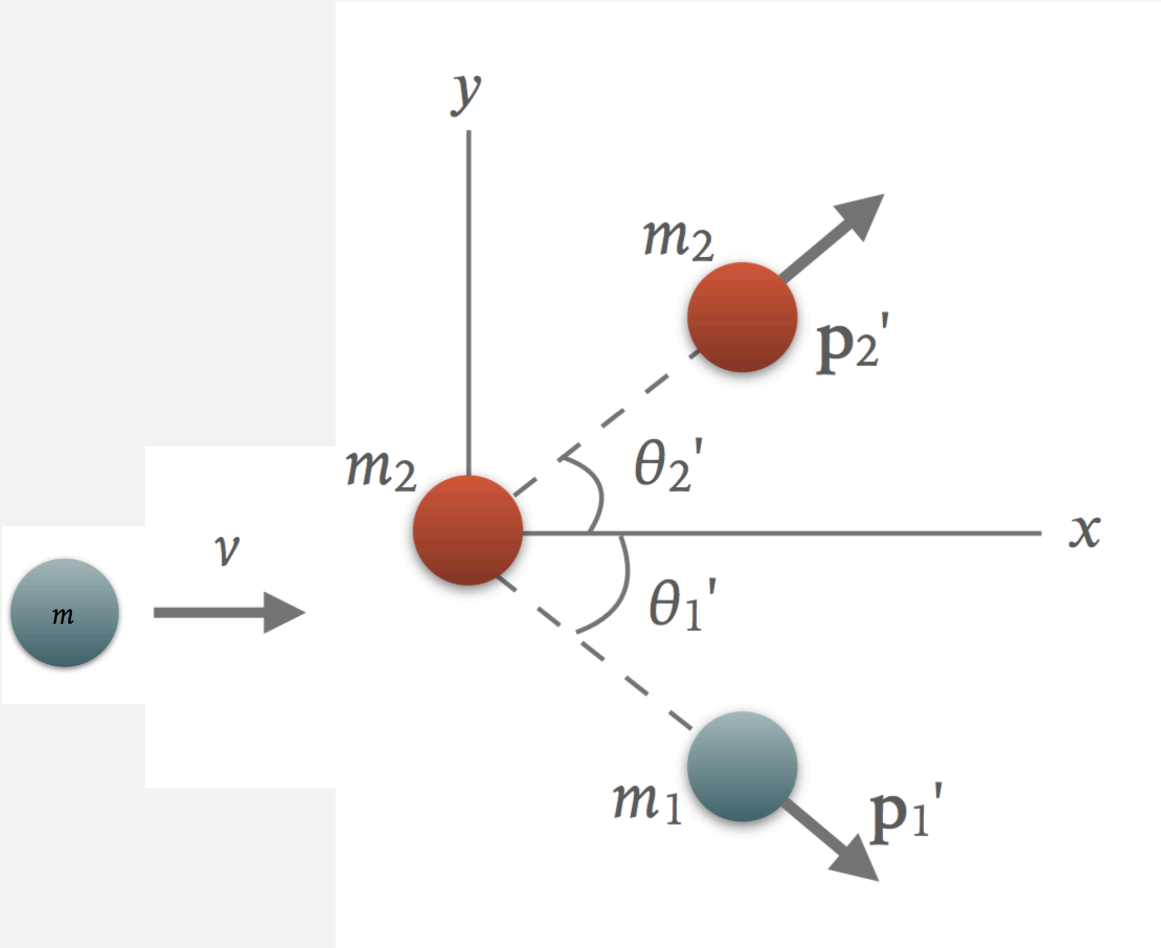
- Both 2.1 m/s
- What is the change in kinetic energy?



2D OR 3D COLLISIONS



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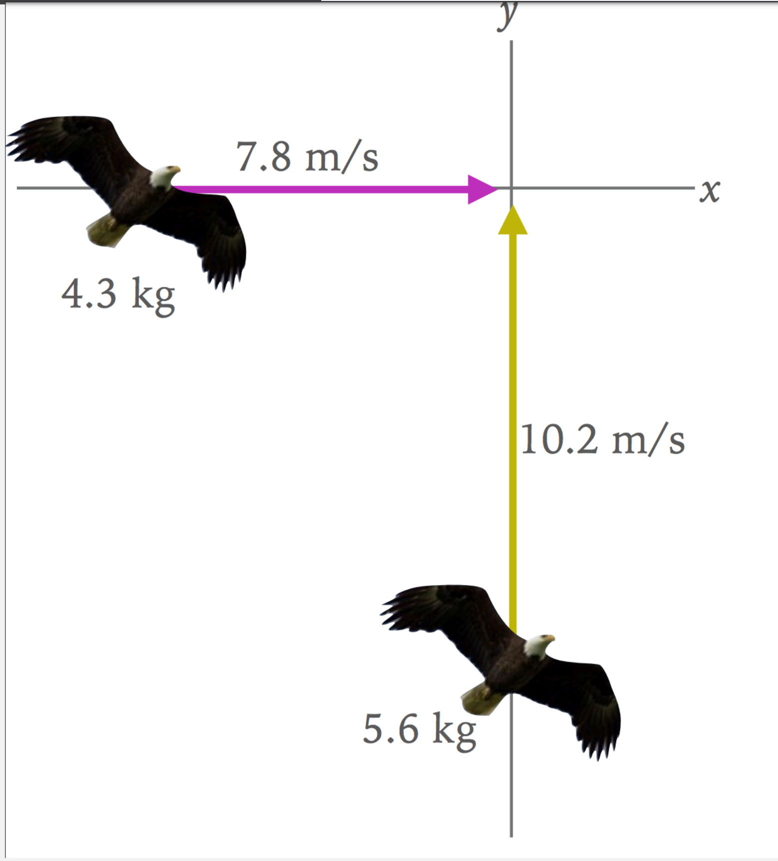

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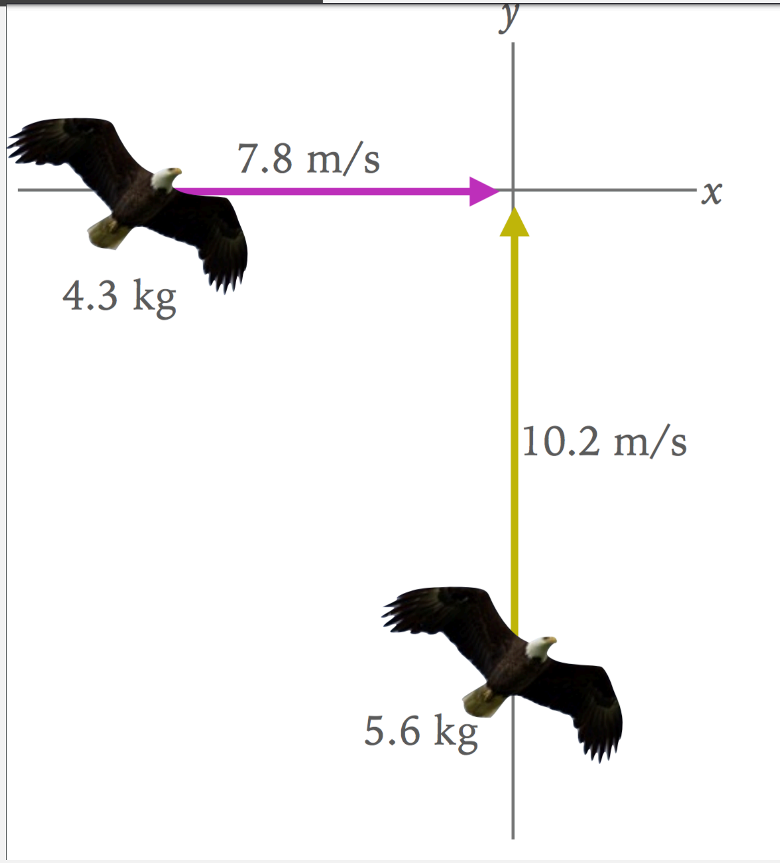
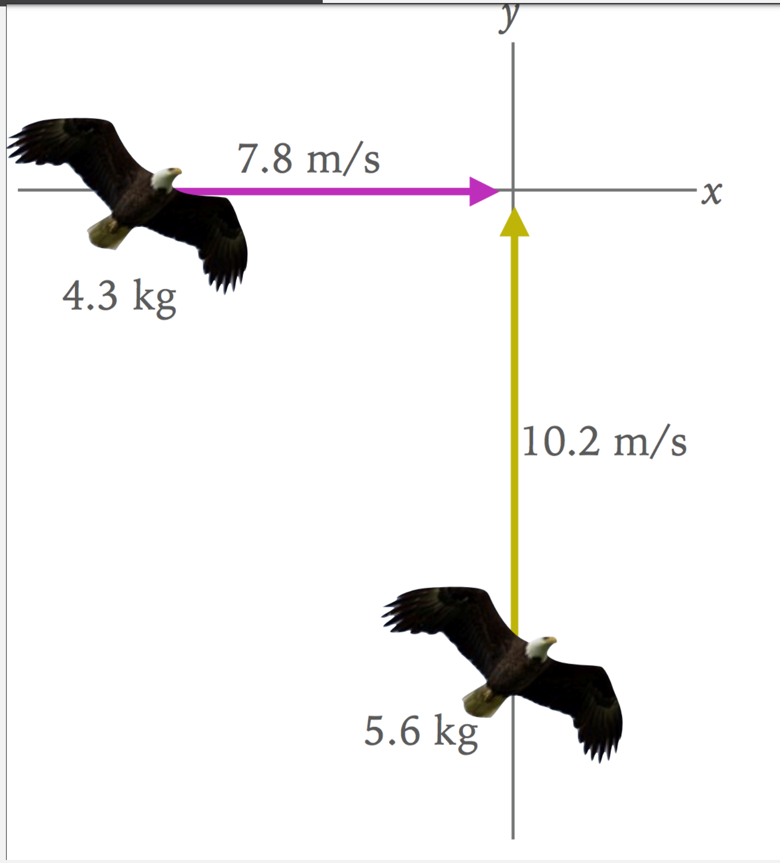


2D OR 3D INELASTIC COLLISIONS

- What is the net work done by nonconservative forces on the system as a result of the collision?
- $W = \Delta KE$
- -200 J



SUSPEND YOUR DISBELIEF A MOMENT- A 175 G FRISBEE MOVING HORIZONTALLY AT 14 M/S EXPLODES IN MIDAIR. A 50. G PIECE CONTINUES IN THE SAME HORIZONTAL DIRECTION AT 20. M/S. A 25 G PIECE DROPS VERTICALLY AT 30 M/S. WHAT MUST BE THE VELOCITY AND DIRECTION (ANGLE) OF THE FINAL 100. G PIECE?

16.3 m/s
27 degrees above horizontal

INITIALLY, THE FRISBEE'S CM IS MOVING AT 14 M/S. HOW FAST IS THE CM MOVING AFTER THE EXPLOSION?